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DIVERSIFYING ISLAMIC HAVEN ASSETS

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ABSTRACT

This study reassesses the safe haven properties of gold and Sukuk using a new framework that incorporates nonstationary volatility and proposes a trading strategy to construct a gold – Sukuk – Islamic equities portfolio that can outperform the hard-to-beat naïve method and the covariance-based approaches. In line with previous studies, it employs data of four exchange-traded funds: Dow Jones Global Sukuk, Wahed FTSE USA Shariah, MSCI Emerging Market Islamic, and SPDR Gold. In the study, an enhanced version of wavelet quintile correlation is proposed to re-evaluate the haven qualities of gold and Sukuk. The results show that gold and Sukuk are safe haven assets. Next, applying a dual momentum strategy, we demonstrate that the risk-adjusted returns of our proposed trading strategy outshine the naïve method and the covariance-based approaches. Our research employs real returns and a rolling window approach to avoid money illusion, overfitting, look-ahead bias, and flawless hindsight. The main results prevail in the robustness tests.

Keywords: Dual momentum, Quantile correlation, Wavelet.
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I. INTRODUCTION

Previous studies have examined the quality of gold and bonds as haven assets. While the view that gold and bonds are haven assets has been well supported by numerous studies, adverse shocks as severe as the COVID-19 pandemic and the Russia-Ukraine war have placed this view into doubt . In this spirit, this research revisits the hypothesis of gold and Islamic bonds or Sukuk as haven assets using a new framework and trading strategy.

Kumar & Padakandla (2022) develop a wavelet quintile correlation that can show the haven quality of gold and Sukuk on different trading days and quintiles. However, the method relies on a single regime volatility. Financial assets contain structural breaks that contribute to variations in volatility, and disregarding this feature can significantly reduce the accuracy of volatility estimates (Ardia et al., 2018). Hence, this study's first and primary objective is to re-evaluate the haven quality of gold and Sukuk using an enhanced version of a wavelet quintile correlation approach.

Once we know the haven quality of gold and Sukuk on different trading days and quintiles, the next discussion is how to allocate the haven assets into a portfolio of Islamic equities. This is also important since many ways exist to construct an optimal gold – Sukuk – Islamic equities portfolio. Therefore, the second objective is to propose a trading strategy that can outperform a hard-to-beat naïve strategy and covariance-based methods widely used in the literature.

The current paper extends the study of Kumar & Padakandla (2022) to re-examine the role of gold and Sukuk in Islamic equities portfolio, whether they are haven assets. However, there are two important differences between this research and that of Kumar & Padakandla (2022). First, this research uses real returns. The objective of using inflation-adjusted returns is to avoid money illusion. Second, the current research examines the role of gold and Sukuk in Islamic equities portfolios, taking into consideration of nonstationary volatility. In addition to the above two, the current study is also interested in understanding the composition of gold – Sukuk – Islamic equities in a portfolio, and this is not a trivial issue. Hence, the current research should also be seen as extending the work of Vliet & Lohre (2023) to gold – Sukuk – Islamic equities portfolio. However, this paper uses dynamic weights instead of the constant weight strategy used by Vliet & Lohre (2023). To my knowledge, the introduction of nonstationary volatility on wavelet quantile correlation and dynamic weight approach on gold – Sukuk – Islamic equities portfolio has not been explored.

The new approaches offer several fresh insights. First, gold and Sukuk are haven assets. Second, the evidence of a safe-haven status is also associated with portfolio allocation; a portfolio's combination of gold – Sukuk – Islamic equities produces significantly lower risk than all Islamic equities portfolios. Finally, a dynamic weight strategy using a dual momentum approach is better than a constant weight strategy. When volatility is very high, stopping loss by converting assets to cash is necessary.

The article's structure is as follows: The literature review begins with the definition of a haven asset and then discusses the empirical research on the abilities of gold and Sukuk to reduce risk. The third section is the methodology, followed by results and analysis. Lastly, I conclude and recommend future research and policies.

II. LITERATURE REVIEW

This section reviews the literature on gold and Sukuk's ability to reduce equities risk. It also deliberates the limitations of previous research that this research addresses.

Baur & Lucey (2010) apply a quantile regression approach and find that gold is a haven asset for equities and non-Islamic bonds in the US, UK, and Germany, both in normal and extreme market conditions. Likewise, applying a similar approach, Baur & McDermott (2010) find that gold is a haven asset for most equities in the developed markets, while gold is not a haven for equities in emerging markets. In addition, Robiyanto et al. (2020) implement a quantile regression approach and conclude that gold is a haven asset for investors concerned about ethics in Indonesia.

In the literature, GARCH-based models are extensively used to evaluate the haven properties of gold. Izadi & Hassan (2018) find that gold futures minimize the risk of the equities portfolio. Similarly, Raza et al. (2019) showed that gold is more effective in hedging Islamic equities than non-Islamic equities. Additionally, Akhtaruzzaman et al. (2021) indicate that gold is a haven asset at the beginning phases of COVID-19. Further, Salisu et al. (2021) indicate that adding gold to an equities portfolio improves the risk-adjusted returns. Bahloul et al. (2023) indicated that gold is a strong haven in developed markets.

Only a few papers conclude that gold is not a haven asset. For example, Bandhu Majumder (2022) concludes that gold is not a haven asset for Indian equities. Furthermore, Corbet et al. (2020) indicate that gold does not function as a haven; instead, it acts as a booster of contagion.

In addition, Ghaemi Asl & Rashidi (2021) employ VAR-BEKK-GARCH to examine the safe-haven features of Sukuk in Middle Eastern and North African countries (MENA). They find that Sukuk is a haven asset. Similarly, Shahzad et al. (2019), utilizing a regime-switching copula technique, find that Sukuk is an excellent haven asset. They propose a portfolio comprising 50% Sukuk and 50% Islamic shares. When there is increased uncertainty, risk-averse investors shift their investment to Sukuk.

The literature on constructing a gold—Sukuk – Islamic equities portfolio is non-existent. The nearest study is Vliet & Lohre (2023). They construct gold—bonds — and non-Islamic equities portfolios and argue that gold is a volatile asset. Hence, the optimal weight of gold in the portfolio is around 10 %, which is determined based on from a constant-weight strategy.

The summary of the selected literature is in **Table 1**. The table shows that previous researchers use various econometric models to evaluate the safe-haven status of Gold, Sukuk, and Islamic equities: a vine-copula framework, dynamic GARCH families, wavelet analysis, Markov-switching copula, wavelet-based quintile, and quantile regression. Kumar & Padakandla (2022) recently propose a new approach, namely theWavelet Quantile Correlation or WQC, to cope with the weaknesses of the previously stated models. Simply put, the WQC can identify safe-haven characteristics over various trading timeframes. However, the WQC proposed by Kumar & Padakandla (2022) does not consider nonstationary volatility.

Therefore, there is a weakness in the original WQC approach. The volatility pattern of financial returns can vary due to shocks such as COVID-19 and the Russia – Ukraine conflict. The conventional GARCH models however mostly assume stationary. Therefore, this research evaluates gold and Sukuk's safe–haven properties based on nonstationary volatility. Two relatively new methods to assess the volatility pattern of financial returns are the MarkovSwitching GARCH or MSGARCH and the Time-Varying GARCH or TV–GARCH (Ardia, Bluteau, Boudt, et al., 2019; Campos-Martins & Sucarrat, 2024). Our research adds the development of the methods used to evaluate the haven properties of gold and Sukuk, offering a new framework. Further, the advantages of gold – Sukuk – Islamic equities portfolio are still unknown. Thus, addressing these issues is the gap that this research fills.

Table 1.
A Selected Review of the Literature

Author(s)	Methods, period of study, and sample	Result	Limitations of the study
Gold as a haven asset			
Baur & Lucey (2010)	Quintile regression. Period of the study: 1995 to 2005. Sample: the US, the UK, German equities, and non-Islamic Bonds.	Gold is a haven asset for equities and non-Islamic Bonds in the US, UK, and Germany, both on normal and during extreme market conditions.	Notably, other researchers have highly cited this research concerning the definition of safe-haven, hedge, and diversifier. However, the authors do not account for regime-switching volatility or inflation without a trading strategy.
Bandhu Majumder (2022)	Various Vector Auto Regression (VAR) – BEKK – GARH models. Period of the study: Dec 2010 to Dec 2020. Sample: Indian stock market.	Gold is not a haven asset for the Indian equities.	The author do not account for inflation.
Izadi & Hassan (2018)	Dynamic conditional correlation – GARCH. Period of the study: Jan 2000 to Oct 2014. Sample: commodity and equity markets of G7 nations.	Gold futures minimize the risk of the equities portfolio.	The authors do not account for regime-switching volatility or inflation and were without a trading strategy.
Raza et al. (2019)	DCC, ADCC, and GO-GARCH. Period of the study: 1996–2015. Sample: Islamic and non-Islamic equities from Dow Jones.	Gold is more effective in hedging Islamic equities than non-Islamic equities.	The authors do not account for regime-switching volatility or inflation and were without a trading strategy.

Table 1.
A Selected Review of the Literature (Continued)

Author(s)	Methods, period of study, and sample	Result	Limitations of the study
Nugroho (2022)	A combination of Markov-switching GARCH – Copula. Period of the study: Dec 2015 to June 2021. Sample: Islamic and non-Islamic indices.	Gold improves the value-at-risk of gold - the Islamic equities portfolio more than the non-Islamic equities portfolio.	Although the author applies a Markov-switching GARCH to account for regime changes, the author utilizes a nominal return without a trading strategy.
Corbet et al. (2020)	DCC – GARCH. Period of the study: March 2019 to March 2020. Sample: hourly data of China's equity market, gold, and Bitcoin.	Gold does not function as a haven; instead, it acts as a booster of contagion.	The authors use nominal returns and do not account for regime-switching volatility, which is very persistent in the early phase of COVID-19.
Salisu, Vo, & Lucey (2021)	Optimal weights and hedge ratios based on VARMA – GARCH. Period of the study: Jan 2019 to Jul 2020. Sample: the US sectoral indices and gold.	Adding gold to an equities portfolio improves the risk-adjusted returns.	The hedge ratios involved short–selling. Conventional short–selling is not allowed by Islamic standards. The authors also use nominal returns and assume no transaction costs for the trading strategy.
Baur & McDermott (2010)	Quintile regression. Period of the study: 1995 to 2005. Sample: non-Islamic equities of G7 nations, BRICS, Australia, and Switzerland.	Gold is a haven asset for most equities in developed markets but not for equities in emerging nations.	The authors use nominal returns and make no recommendation regarding the percentage of gold in a portfolio.
Akhtaruzzaman et al. (2021)	Optimal weights and hedge ratios based on DCC – GARCH and the quintile regression. Period of the study: Dec 2019 to Apr 2020. Sample: hourly returns of gold and conventional equity indices of the USA, Europe, Japan, and China.	Gold is a haven asset at the very beginning phases of COVID-19.	The recommended optimal weight of gold in the portfolio is 56%. Still, it is not economical for investors to rebalance their portfolio hourly. The authors also use nominal returns and assume no transaction costs.
Baur & Smales (2020)	Linear and GARCH-based regression. Period of the study: Jan 1985 to Oct 2018. Sample: precious metals futures and S&P 500 futures.	Gold acts as a consistent haven asset against geopolitical risk.	The authors use the signal of the geopolitical risk index as a trading strategy without considering transaction costs. The daily and monthly signals may generate high transaction costs.

Table 1.
A Selected Review of the Literature (Continued)

Author(s)	Methods, period of study, and sample	Result	Limitations of the study
Salisu, Vo, & Lawal (2021)	Optimal portfolio weights based on VARMA–GARCH. Period of the study: Jan 2016 to Aug 2020. Sample: precious metals and crude oil.	Gold serves as a haven asset towards the oil price risk.	The optimal portfolio weight strategy is outdated and impractical since it involves daily rebalancing (implying high transaction costs). The authors also use nominal returns.
Bahloul et al. (2023)	DCC-GARCH-based regression of Ratner & Chiu (2013). Period of the study: May 2015 to May 2020. Sample: Gold, Islamic, and non-Islamic equities from the US, Italy, Russia, France, the UK, Malaysia, Spain, China, Germany, and Brazil.	Gold serves as a strong haven in developed markets. Interestingly, Islamic equities act as weak haven assets against the risk of non-Islamic equities in Spain, France, the UK, and Malaysia.	The authors use nominal returns, do not account for regime-switching volatility, offer no trading strategy, and assume no transaction cost. Simply put, the authors do not recommend the proportion of gold in a portfolio.
Vliet & Lohre (2023)	Constant weights (10% Gold, 45% conventional bonds, 45% low-volatility equities) with yearly rebalancing. Period of the study: 1975 to 2022. Sample: bonds and equities in the US.	The inclusion of gold in a typical bond–equities portfolio reduces risk.	This research only focuses on the US market with lower inflation rates than emerging markets in 2020 - 2022.
Kumar & Padakandla (2022)	A newly developed method (wavelet quintile correlation). Period of the study: Jan 2015 to Dec 2020. Sample: non-Islamic equities from developed markets (France, the US, India, and Europe), gold, and Bitcoin.	Gold serves as a haven asset.	The authors use nominal returns and do not account for regime-switching volatility.
Rusmita et al. (2024)	A threshold GARCH approach (TGARCH) and a quintile regression. Period of the study: Jan 2011 to Oct 2022. Sample: Antam gold and the Jakarta Islamic Index.	Gold is a haven asset.	The authors use nominal returns and do not account for regime-switching volatility.
Sukuk as a diversifier			
Nugroho & Kusumawardhani (2023)	A novel hedge ratio involving a modified EWMA approach, DECO-GARCH, and wavelet. Period of the study: Jan 2020 to Oct 2023. Sample: Islamic exchange-traded funds.	Sukuk acts as a diversifier.	The authors use nominal returns and do not account for regime-switching volatility, making the trading strategy from the hedge ratio approach impractical.

Table 1.
A Selected Review of the Literature (Continued)

Author(s)	Methods, period of study, and sample	Result	Limitations of the study
Naeem et al. (2023)	Asymmetric DCC-GARCH regression of Ratner & Chiu (2013) and hedge ratio. Period of the study: Jan 2020 to Oct 2023. Sample: Dow Jones Sukuk Bond Index, green bonds, and equities from ten nations.	Sukuk acts as a diversifier.	The limitations of this study are similar to those in Nugroho & Kusumawardhani (2023).
Qadri et al. (2024)	GARCH-based regression. Period of the study: Aug 2012 to Jun 2022. Sample: sukuk and conventional bond indices from various countries.	Sukuk is not a haven asset.	The authors use nominal returns and do not account for regime-switching volatility.

III. METHODOLOGY

3.1. Data

This study uses the following exchange-traded funds (ETFs): Dow Jones Global Sukuk, FTSE USA Shariah, iShares MSCI EM Islamic, and SPDR Gold. The underlying assets for Sukuk ETF include KSA Sukuk Limited, issued by Saudi Arabia, with a profit rate of 3.628%. The average maturity of the Islamic bonds in the ETF is about eight years. The daily data covers the sample from Jan 02, 2020, to Dec 29, 2023. This sample period is dictated by data availability. The data are sourced from https://www.tiingo.com/, which is a reliable data source and used by others (Liu et al., 2023). The returns are calculated by taking a natural log of today's price divided by yesterday's price.

In the literature, few studies adjust nominal returns with inflation. Along this line, this research uses real returns. The inflation data for the USA market are from fred.stlouisfed.org while those for the emerging markets are from www.imf.org. All prices are in US dollars to eliminate bias from foreign exchange fluctuations.

This research utilizes those ETFs for the following reasons: First, previous studies concluded that Sukuk are not safe-haven assets (see **Table 1**). However, prior results do not consider dual-regime volatility. Hence, this study compares the performance of a previous model and our model using the same data set. Second, ETFs can offer lower operating costs than traditional open-end funds, flexible trading, and greater transparency.

3.2. Nonstationarity Volatility Models – Wavelet Quintile Correlation (MSGARCH-WQC and TVGARCH-WQC)

The conventional WQC does not account for nonstationary volatility. Financial assets contain structural breaks that contribute to their volatility movement, and disregarding this feature can significantly reduce the accuracy of volatility estimates (Ardia et al., 2018). TV-GARCH is more robust when modeling nonstationary volatility (Campos-Martins & Sucarrat, 2024). Hence, this study proposes MSGARCH-WQC and TVGARCH-WQC.

3.2.1 Markov-switching GARCH (MSGARCH)

This study follows a two-step methodology to estimate MSGARCH-WQC. First, we utilize MSGARCH and an AR (1) filter to exclude autoregressive influences from the data before estimating the models based on the residuals (Ardia, Bluteau, & Rüede, 2019). The improved risk estimates of the model stem from their ability to adjust to changes in the unconditional volatility level swiftly. MSGARCH can be estimated using the Maximum Likelihood method. Nonetheless, several current studies show certain benefits to using a Bayesian approach (Ardia, Kolly, et al., 2017; Casarin et al., 2024). For instance, the Bayesian approach allows investigatation of the joint posterior distribution of the model parameters. Hence, we apply a Bayesian technique to estimate the model parameters using Markov chain Monte Carlo (MCMC) simulations.

In addition, this study incorporates E-GARCH for the scedastic specification (Nelson, 1991). We use dual regimes ($\mathcal{K}$ = 2). Compared to symmetric GARCH, the E-GARCH, which is an asymmetric GARCH, fits data better. Haas, Mittnik, & Paolella (2004) incorporate the scedastic specification into the MSGARCH framework. Still, the model is more robust by permitting innovations to come from distributions other than normal (Cerqueti et al., 2020). For the innovations, we thus use the Student-t and the Skewed Student-t. To save space, the interested reader is referred to Ardia, Bluteau, Boudt, et al. (2019) to estimate MSGARCH using the *R* statistical program.

3.2.2 Time-varying GARCH (TVGARCH)

This section briefly explains the test of nonstationary GARCH. Since the MSGARCH is conducted on univariate settings, the TV–GARCH approach is implemented similarly. Verifying whether the unconditional variance is time-invariant is crucial before assessing a TV-GARCH model. Rejecting the null hypothesis (H_o) indicates nonstationarity, suggesting that a conventional GARCH model with constant parameters is not appropriate and consequently not suited to fit the data. The unconditional variance under the alternative hypothesis is time-varying.

$$g_t^* = \vartheta_0^* + s_t\vartheta_1^* + s_t^2\vartheta_2^* + s_t^2\vartheta_3^*$$

where ϑ_0^*, ϑ_1^*, ϑ_2^*, and ϑ_3^* are functions of the initial parameters; g_t^* is the number of transitions that need to be identified. H_o holds if $\vartheta_1^* = \vartheta_2^* = \vartheta_3^* = 0$. To save space, the interested reader is referred to Campos-Martins & Sucarrat (2024) to estimate TVGARCH using the *R* statistical program.

3.2.3. Wavelet Quintile Correlation (WQC)

We obtain WQC with the following steps. First, we receive the Quantile Correlation (QC) of two variables $\widetilde{\mathcal{X}}$ and $\widetilde{\mathcal{Y}}$ based on Li et al. (2015). We briefly discuss QC below.

Let $\tilde{\mathcal{Q}}_{\tau,\widetilde{\mathcal{X}}}$ be the τ^{th} quantile of $\widetilde{\mathcal{X}}$ and $\tilde{\mathcal{Q}}_{\tau,\tilde{\mathcal{Y}}}(\widetilde{\mathcal{X}})$ be the τ^{th} quantile of $\widetilde{\mathcal{Y}}$ conditional upon $\widetilde{\mathcal{X}}$. $\tilde{\mathcal{Q}}_{\tau,\tilde{\mathcal{Y}}}(\widetilde{\mathcal{X}})$ is independent of $\widetilde{\mathcal{X}}$ if and only if the random variables *I* ($\tilde{\mathcal{Y}}$ - $\tilde{\mathcal{Q}}_{\tau,\tilde{\mathcal{Y}}}$) > 0 and $\widetilde{\mathcal{X}}$ are independent. *I*(.) is the indicator function. For $0 < \tau < 1$, the quantile covariance is*:*

$$qcov_\tau(\tilde{\mathcal{Y}},\widetilde{\mathcal{X}}) = cov\{I(\tilde{\mathcal{Y}} - \tilde{\mathcal{Q}}_{\tau,\tilde{\mathcal{Y}}} > 0), x\} = \hat{\text{E}}(\varphi_\tau(\tilde{\mathcal{Y}} - \tilde{\mathcal{Q}}_{\tau,\tilde{\mathcal{Y}}})(\widetilde{\mathcal{X}} - \hat{\text{E}}(\widetilde{\mathcal{X}})) \quad (1)$$

$\varphi_\tau(\widetilde{w}) = \tau - I(\widetilde{w} < 0)$. The QC is:

$$qcor_t(\widetilde{\mathcal{X}},\tilde{\mathcal{Y}}) = \frac{qcov_\tau(\tilde{\mathcal{Y}},\mathcal{X})}{\sqrt{(var(\theta_\tau(\tilde{\mathcal{Y}}-\mathcal{Q}_{\tau,\tilde{\mathcal{Y}}})var(\widetilde{\mathcal{X}})}} \quad (2)$$

The QC shows the correlation between asset pairs across different quantiles. An asset with safe-haven qualities should negatively correlate with another asset in a turbulent market, which QC will identify in the lower quantiles.

Additionally, we assume that investors have different preferences over different time horizons when selecting a safe-haven asset. We derive these dynamics by looking at the dependence structure over multiple timescales. As a result, this study utilizes Wavelet Quantile Correlation (WQC). We use a *maximum overlapping discrete wavelet transform* (*MODWT*), as Percival & Walden (2000) suggest, to decompose the asset pairs. $\widetilde{\mathcal{X}}_t$ and $\tilde{\mathcal{Y}}_t$. we briefly discuss *MODWT* below.

Let $\widetilde{\mathcal{X}}[\tilde{\imath}]$ be a signal of length *T*, such that $T=2^J$ for some integer $\mathcal{J}$. An orthogonal wavelet defines the low-pass and high-pass filters, $\widetilde{\hbar}_1[\tilde{\imath}]$ and $\widetilde{g}_1[\tilde{\imath}]$, respectively. To produce the approximation coefficients $\tilde{a}_1[\tilde{\imath}]$ and $\tilde{d}_1[\tilde{\imath}]$, $\widetilde{\mathcal{X}}[\tilde{\imath}]$ is convolved with $\widetilde{\hbar}_1[\tilde{\imath}]$ at the first level and with $\widetilde{g}_1[\tilde{\imath}]$ at the second.

$$\tilde{a}_1[\tilde{\imath}] = \widetilde{\hbar}_1[\tilde{\imath}] * s[\tilde{\imath}] = \textstyle\sum_k \widetilde{\hbar}_1[\tilde{\imath} - \tilde{k}]s[k] \quad (3)$$

$$\tilde{d}_1[\tilde{\imath}] = \widetilde{g}_1[\tilde{\imath}] * s[\tilde{\imath}] = \textstyle\sum_k \widetilde{g}_1[\tilde{\imath} - \tilde{k}]s[k] \quad (4)$$

Next, we apply the same strategy to filter $\tilde{a}_1[\tilde{\imath}]$, but we utilize modified filters $\widetilde{\hbar}_2[\tilde{\imath}]$ and $\widetilde{g}_2[\tilde{\imath}]$, which we get by dyadic up-sampling $\widetilde{\hbar}_1[\tilde{\imath}]$ and $\widetilde{g}_1[\tilde{\imath}]$. For $\boldsymbol{\mathcal{J}} = 1, 2.... \mathcal{J}_0 - 1$, where $\mathcal{J}_0 \leq \boldsymbol{\mathcal{J}}$, the coefficients are computed as follows:

$$\tilde{a}_{\mathcal{J}+1}[\tilde{\imath}] = \widetilde{\hbar}_{\mathcal{J}+1}[\tilde{\imath}] * \tilde{a}_{\mathcal{J}}[\tilde{\imath}] = \textstyle\sum_k \widetilde{\hbar}_{\mathcal{J}+1}[\tilde{\imath} - \hat{k}]\tilde{a}_{\mathcal{J}}[\tilde{\imath}] \quad (5)$$

$$\tilde{d}_{\mathcal{J}+1}[\tilde{\imath}] = \widetilde{g}_{\mathcal{J}+1}[\tilde{\imath}] * \tilde{a}_{\mathcal{J}}[\tilde{\imath}] = \textstyle\sum_k \widetilde{g}_{\mathcal{J}+1}[\tilde{n} - \hat{k}]\tilde{a}_{\mathcal{J}}[\mathcal{J}] \quad (6)$$

$\tilde{h}_{\mathcal{J}+1}[\tilde{\imath}] = \tilde{U}(\tilde{h}_{\mathcal{J}}[\tilde{\imath}])$ and $\tilde{g}_{\mathcal{J}+1}[\tilde{\imath}] = \tilde{U}(\tilde{g}_{\mathcal{J}}[\tilde{\imath}])$. $\tilde{\mathcal{U}}$ is the up-sampling operator. After applying a $\boldsymbol{\mathcal{J}}$ level decomposition to $\tilde{\mathcal{X}}_t$ and $\tilde{\mathcal{Y}}_t$, the *WQC* is obtained as follows:

$$WQC_{\tau}\left(\tilde{d}_{\mathcal{J}}|\tilde{\mathcal{X}}|, \tilde{d}_{\mathcal{J}}|\tilde{\mathcal{Y}}|\right) = \frac{qcov_{\tau}\left(\tilde{d}_{\mathcal{J}}|\tilde{\mathcal{X}}|, \tilde{d}_{\mathcal{J}}|\tilde{\mathcal{Y}}|\right)}{\sqrt{var\left(\theta_{\tau}\left(\tilde{d}_{\mathcal{J}}|\tilde{\mathcal{Y}}| - Q_{\tau,\tilde{d}_{\mathcal{J}}|\tilde{\mathcal{Y}}|}\right)\right) var\left(\tilde{d}_{\mathcal{J}}|\tilde{\mathcal{X}}\right)}} \tag{7}$$

3.3. Portfolio Construction Methods

3.3.1. Naïve (The Benchmark)

The first method for portfolio optimization used in this study is 1/N or naïve strategy. DeMiguel et al. (2009) illustrate that errors in measuring means and covariances undermined all the benefits from optimal diversification instead of naive diversification.

3.3.2. Minimum Variance

The calculation of the minimum-variance weight is as follows (Ardia et al., 2017):

$$w_{min} \equiv \underset{w \in C}{argmin} \{\acute{w} \textstyle\sum w\} \tag{8}$$

where $\mathcal{C} \equiv \left\{w \in \mathcal{R}_{+}^{\mathcal{N}} \mid \acute{w}\; l\, \mathcal{N} = 1\right\}$ is the long-only investment constraint.

3.3.3. Risk Parity

All of the assets in an equal-risk-contribution portfolio contribute equally to the total volatility of the portfolio. In other words, it's the portfolio where each asset's percentage contribution to volatility risk equals 1/N. The calculation of the portfolio weight is as follows (Maillard et al., 2010):

$$w_{erc} \equiv \underset{w \in C}{argmin} \left\{\textstyle\sum_{i=1}^{\mathcal{N}} (\%\mathcal{RC}_i - \frac{1}{N})^2\right\} \tag{9}$$

Where $\%\mathcal{RC}_i \equiv \frac{w_i[\sum w]_i}{w \acute{\sum} w}$.

3.3.4. Dual Momentum

Dual momentum assigns a relative value to each asset class based on how well it has performed over the past three months relative to other assets in the same class and whether or not it has had a positive return. As long as the top-performing asset in the asset class has a positive return above zero, dual momentum invests in those assets. Otherwise, the allocation is shifted to cash. This strategy is inspired by Antonacci (2012).

3.4. Rolling Window Approach

We run the portfolio construction methods in a rolling window approach to avoid overfitting, look-ahead bias, and ideal hindsight. Furthermore, we intentionally make our analysis straightforward and non-exclusive. Our fundamental rationale for taking this method is to verify that the strategies presented are practical for investors.

Hence, this research uses the following steps to generate the portfolio weights: 1) To determine the parameters in the optimization, we use the closest M=250 days' data before the rebalancing time *t*; 2) Solve the relevant optimization problem to obtain the weights at *t*; 3) Following the holding period τ (also known as the rebalancing period), rebalance the weights by carrying out procedures 1) and 2).

IV. RESULTS AND ANALYSIS

4.1. Descriptive Statistics

Table 2 presents descriptive statistics of the variables. This study adjusts nominal returns with inflation. The table implies that inflation in the emerging nations is higher, indicating that, by the drawdown, an investment's peak-to-trough decreases over a given period. Inflation also affects the risk-adjusted returns (Sharpe, Sortino, and Omega). The table also reveals that the kurtosis values are greater than 3, implying a leptokurtic distribution.

Interestingly, Sukuk has a positive skewness. Skewness is the third central moment, commonly used to assess the distribution's divergence from symmetry. Assets with a positive skew typically have minor losses and few significant gains. On the contrary, negatively skewed assets usually have many small gains and few significant losses.

4.2. MSGARCH

This study employs dual-regime *E*-GARCH with the Student-t and the Skewed Student-t for the innovations. Our selection is appropriate based on the deviance information criterion (**Table 3**).

Table 4 reports the parameters from *E*-GARCH with the Skewed Student-t distribution up to two regimes. As expected, the unconditional volatility (annualized) or UV of $\mathcal{K}$ = *2* is higher than $\mathcal{K}$ = *1*. In particular, the Islamic equities in the emerging markets have higher volatility. Moreover, the values of $\frac{\omega_{\mathcal{K}}}{(1-\alpha_{\mathcal{K}}+\beta_{\mathcal{K}})}$ in $\mathcal{K}$ = *2*, are higher than in the single-regime ($\mathcal{K}$ = *1*). Specifically, the long-term volatility in the USA market is higher than in the emerging markets. Both regimes are very persistent, with posterior probabilities of ρ_{11} and ρ_{22} higher than 95%. Further, **Figure 1** clearly illustrates that the high volatility regime (red lines) occurred during the early phase of the COVID-19 pandemic and the Russia-Ukraine conflict.

Table 3.
Deviance Information Criterion

	Normal	Student-*t*	Skewed Student-*t*
Emerging Markets			
single-regime			
Standard GARCH	3210.71	3176.35	3176.44
E-GARCH	3204.64	3183.03	3177.08
dual-regime			
Standard GARCH	3187.18	3182.42	3181.63
E-GARCH	3196.37	3170.84	3169.79
USA			
single-regime			
Standard GARCH	3101.24	3068.88	3058.73
E-GARCH	3084.35	3031.34	3020.13
dual-regime			
Standard GARCH	3075.16	3069.31	3067.49
E-GARCH	3043.34	3021.54	3014.85

Notes: The highlighted values show that the dual–regime MSGARCH model outperforms the single–regime model.

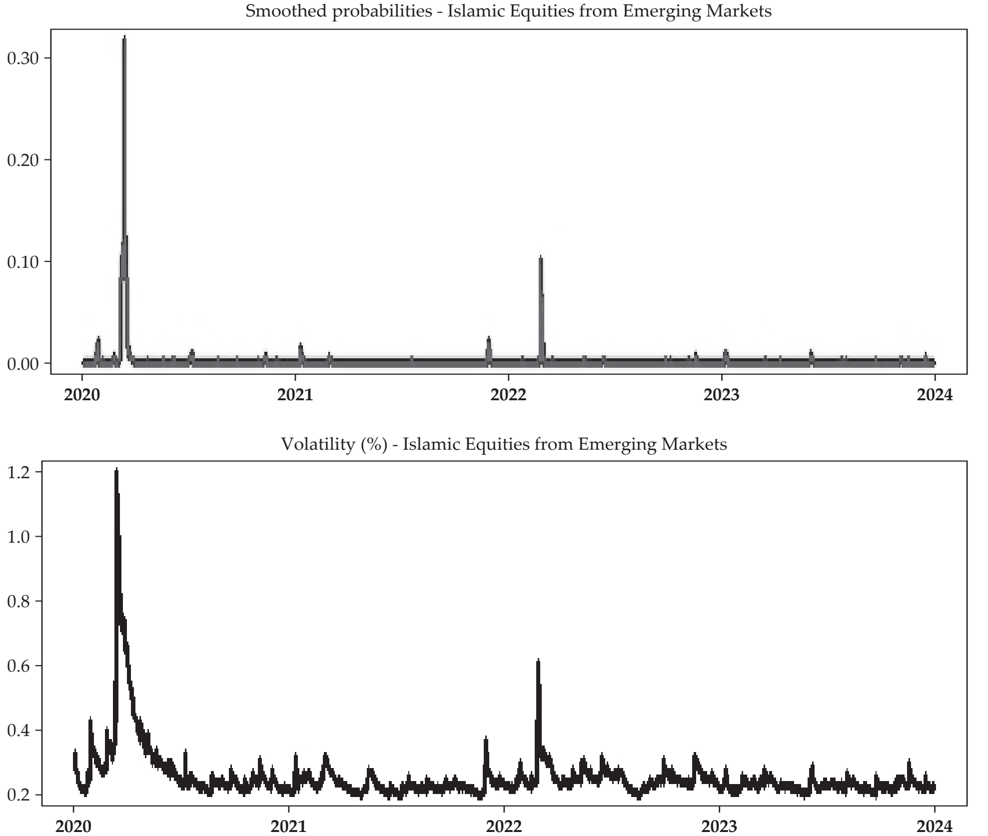


Figure 1.
Smoothed Probabilities and Volatility

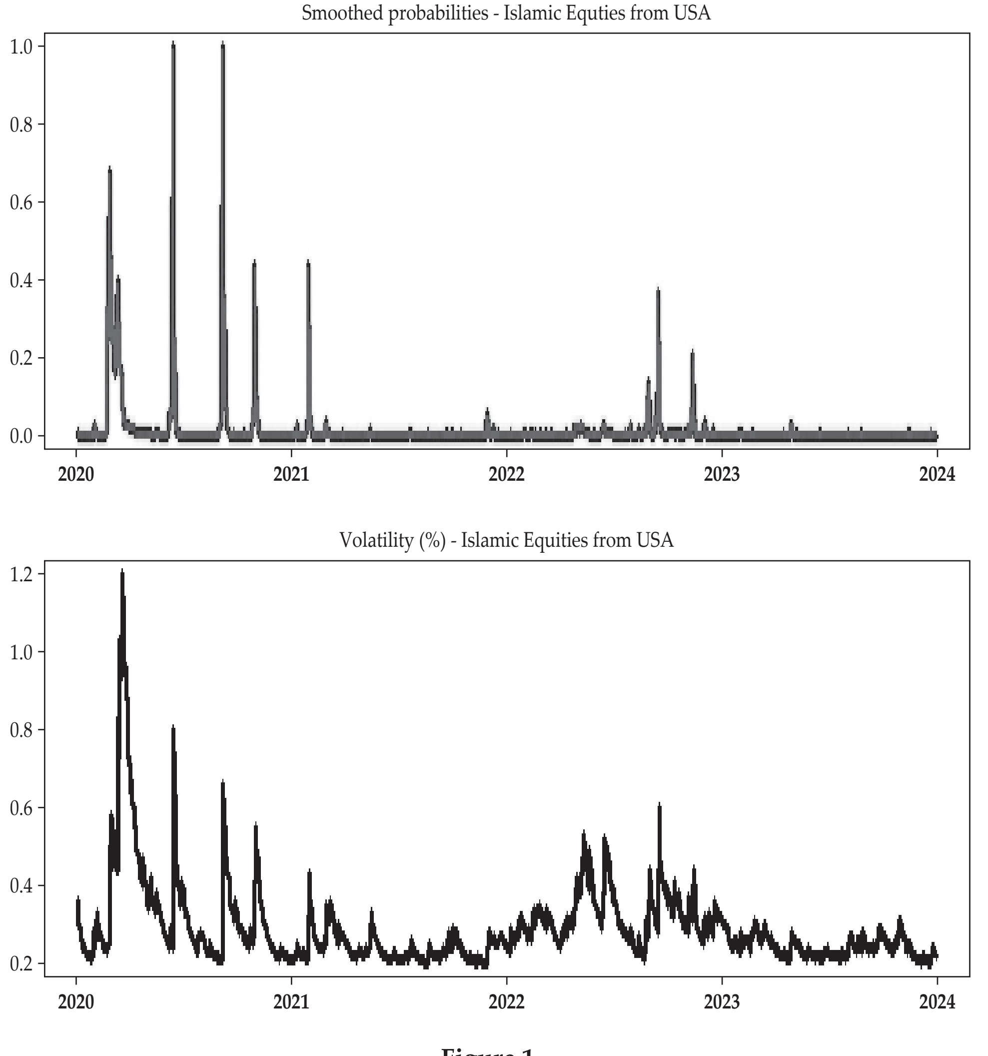


Figure 1.
Smoothed Probabilities and Volatility (Continued)

Table 2.
Descriptive Statistics

	nominal returns		real returns		nominal returns		real returns (emerging markets)		real returns (USA)	
	USA	Emerging	USA	Emerging	Gold	Sukuk	Gold	Sukuk	Gold	Sukuk
Mean returns %	13.585	-1.560	9.055	-8.948	7.561	0.627	0.169	-6.763	2.760	-4.999
Std. deviation %	22.702	22.227	22.705	22.226	15.706	5.413	15.703	5.417	15.668	5.466
Downside Volatility %	1.069	1.042	1.069	1.042	0.722	0.237	0.722	0.237	0.719	0.241
Minimum returns %	-10.892	-13.234	-10.896	-13.252	-5.519	-1.640	-5.539	-1.660	-5.524	-1.695
Maximum returns %	8.053	8.404	8.048	8.382	4.739	2.000	4.717	1.965	4.734	1.983
Skewness	-0.824	-0.952	-0.817	-0.946	-0.315	0.116	-0.311	0.121	-0.305	0.043
Kurtosis	9.232	10.852	9.196	10.822	2.995	4.255	2.979	4.188	2.941	4.391
Sharpe %	2.296	-0.263	1.520	-1.489	1.848	0.481	0.041	-4.905	0.669	-3.599
Sortino %	5.158	-0.592	3.412	-3.354	4.244	1.057	0.093	-10.704	1.534	-7.912
Omega	1.117	0.987	1.077	0.930	1.087	1.021	1.002	0.799	1.031	0.848
Value-at-Risk %	-2.347	-2.357	-2.364	-2.384	-1.624	-0.518	-1.652	-0.547	-1.637	-0.551
Median drawdown %	10.209	8.814	10.627	9.489	5.459	2.805	6.672	7.256	6.506	3.741
Maximum drawdown %	35.369	39.078	35.440	50.283	22.053	12.314	33.376	27.565	32.339	24.419

Notes: This table shows the basic statistics of the return series. Real returns indicate that nominal returns have been adjusted to inflation. Kurtosis values were greater than 3, implying a leptokurtic distribution. A positive skewness (favorable) typically has minor losses and few significant gains. The larger the risk-adjusted returns (Sharpe, Sortino, and Omega), the better the performance. The larger the drawdown, the more substantial the decrease in an investment's peak-to-trough over time and the riskier the investment. Similarly, the larger the downside volatility, measured by semi-deviation, which assesses the below mean fluctuations, the riskier the investment. In addition, the lower the value-at-risk, a loss that we are confident will not be surpassed if the portfolio is held for a certain period, the riskier the investment. The daily data covers the sample from Jan 02, 2020, to Dec 29, 2023.

Table 4.
Parameter Estimates

	single-regime		dual-regime	
	USA	**Emerging**	**USA**	**Emerging**
Regime ($\mathcal{K}$ = 1)				
ω_1	0.015	0.020	0.008	0.023
α_1	0.198	0.186	0.151	0.147
γ_1	-0.138	-0.081	-0.195	-0.101
β_1	0.843	0.943	0.962	0.878
η_1	10.427	17.744	21.020	13.412
ζ_1	0.848	0.919	0.843	0.939
ρ_{11}	1.000	1.000	0.995	0.990
$\mathbb{UV}_1$	17.316	19.998	22.246	18.082
$\frac{\omega_1}{(1-\alpha_1+\beta_1)}$	0.009	0.011	0.004	0.013
Regime ($\mathcal{K}$ = 2)				
ω_2			0.117	0.070
α_2			0.088	0.136
γ_2			-0.104	-0.222
β_2			0.831	0.913
η_2			27.207	11.705
ζ_2			0.572	0.697
ρ_2			0.984	0.979
$\mathbb{UV}_2$			24.757	32.087
$\frac{\omega_2}{(1-\alpha_2+\beta_2)}$			0.067	0.039

Notes: The table shows the mean of the posterior sample for the *E*-GARCH model and *Skewed S*tudent-t with single and dual regimes. The parameters of $\eta_{\mathcal{K}}$ and ζ_2 are the tail and asymmetry values, respectively. The estimates are taken from 1000 draws. $\mathbb{UV}_{\mathcal{K}}$ is the unconditional volatility. *E*-GARCH estimates in regime $\mathcal{K}$ are computed as ln $\hbar_{\mathcal{K},t} = \omega_{\mathcal{K}} + \alpha_{\mathcal{K}}(|y_{t-1}| - E[|y_{t-1}|]) + \gamma_{\mathcal{K}} y_{t-1} + \beta_{\mathcal{K}} \ln(\hbar_{\mathcal{K},t-1})$. The values of $\frac{\omega_{\mathcal{K}}}{(1-\alpha_{\mathcal{K}}+\beta_{\mathcal{K}})}$ indicate long-term volatility.

4.3. The Conventional Wavelet Quintile Correlation (WQC)

If an asset has safe-haven features, its correlation with other assets must be negative during market turmoil, particularly at lower quantiles. **Figure 2** shows that QC is positive across trading periods and lower quintiles for Gold/Islamic equities in emerging markets. QC is also positive across the median quintiles. These results imply that gold is not a haven asset but a diversifier for Islamic equities in emerging markets. Also, **Figure 2** shows that QC is positive across trading periods and lower quintiles for Gold/Islamic equities in the US. However, QC is negative at the median quintiles for one trading year. These results suggest that gold is not a haven asset but a strong hedge for Islamic equities in the US.

Moreover, **Figure 3** shows that QC is positive across trading periods and quintiles for Sukuk/Islamic equities in the US. Similarly, QC is positive across trading periods and quintiles for Sukuk/Islamic equities in emerging markets. These results indicate that Sukuk is a diversifier for Islamic equities in emerging markets and the USA.

4.4. MSGARCH – WQC

After accounting for dual-regime volatility, **Figure 4** illustrates that QC is negative for 32-64 days at the lower quintiles for Gold/Islamic equities in the USA. In addition, QC is also negative across several trading days at the median quintiles. These results imply that gold is a haven asset and a hedge for Islamic equities in the USA.

Moreover, **Figure 4** also illustrates that QC is negative for 32-64 days at the lower quintiles for Gold/Islamic equities in emerging markets. However, QC is positive across the median quintiles. These results imply that gold is a haven asset and a diversifier for Islamic equities in emerging markets.

Further, **Figure 5** indicates that QC is negative for 2-4 days at the lower quintiles for Sukuk/Islamic equities in emerging markets. However, QC is positive across the median quintiles. These results suggest that Sukuk is a haven asset and a diversifier for Islamic equities in emerging markets.

In addition, **Figure 5** shows that QC is positive across trading days at the lower quintiles for Sukuk/Islamic equities in the USA. Also, QC is positive across trading days at the median quintiles. These results suggest that Sukuk is not a haven asset but a diversifier for Islamic equities in the US market.

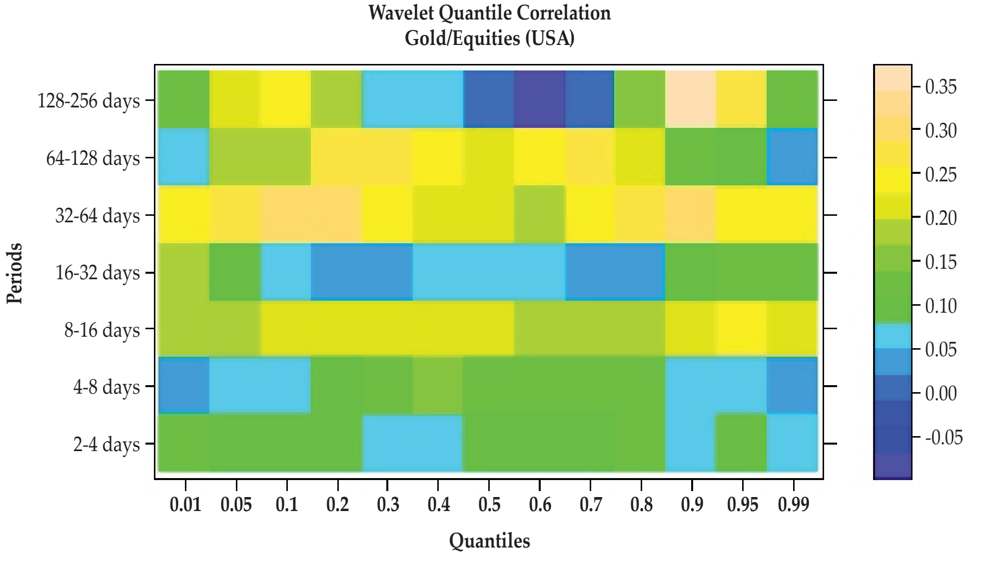


Figure 2.
Wavelet Quintile Correlation (Gold/Equities)

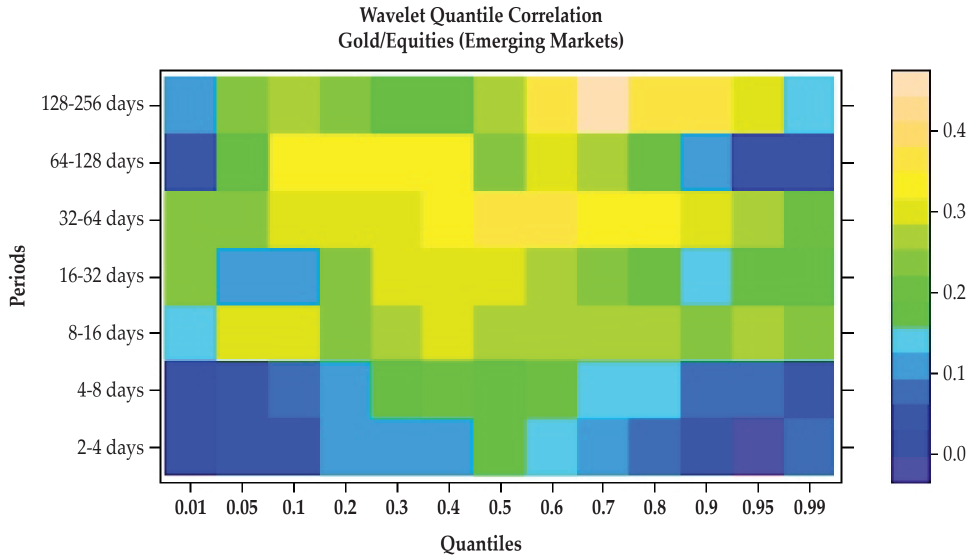


Figure 2.
Wavelet Quintile Correlation (Gold/Equities) (Continued)

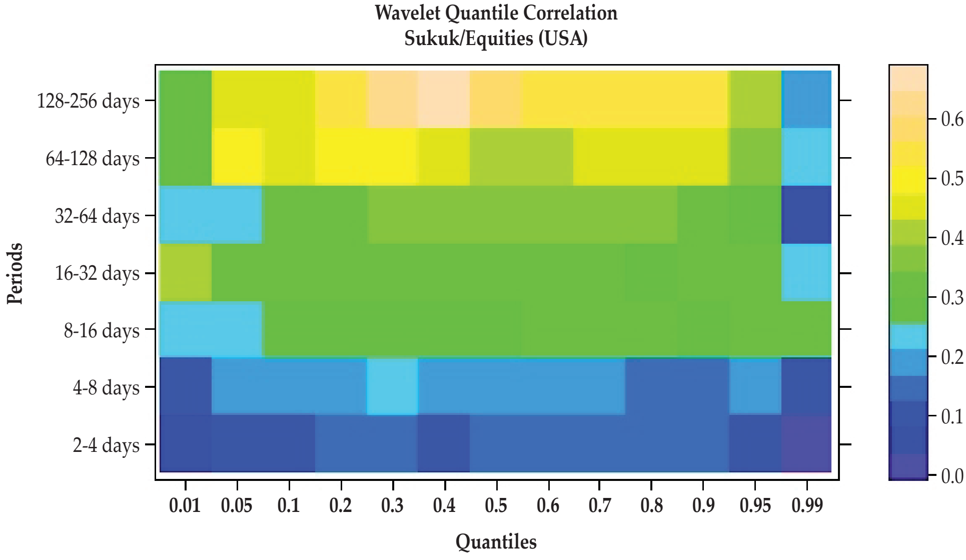


Figure 3.
Wavelet Quintile Correlation (Sukuk/Equities)

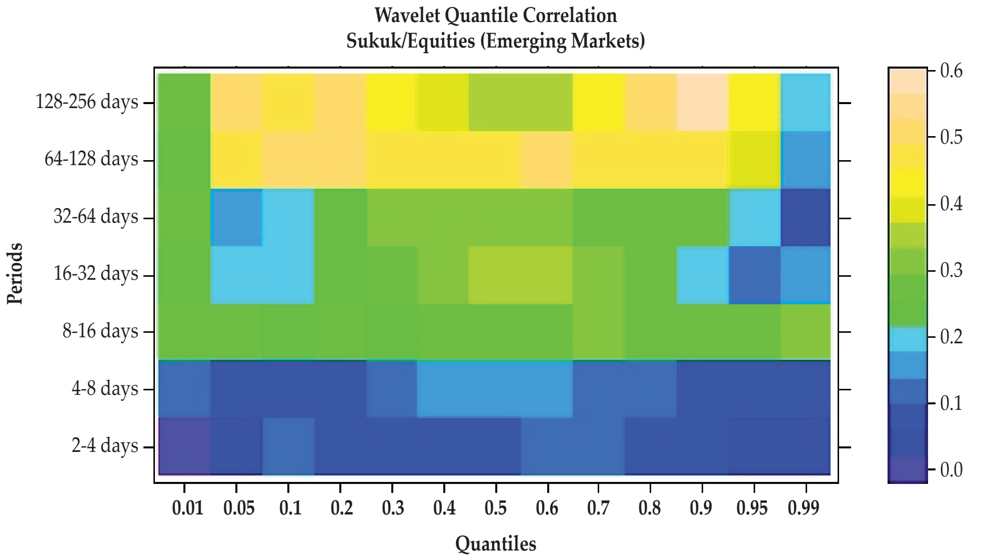


Figure 3.
Wavelet Quintile Correlation (Sukuk/Equities) (Continued)

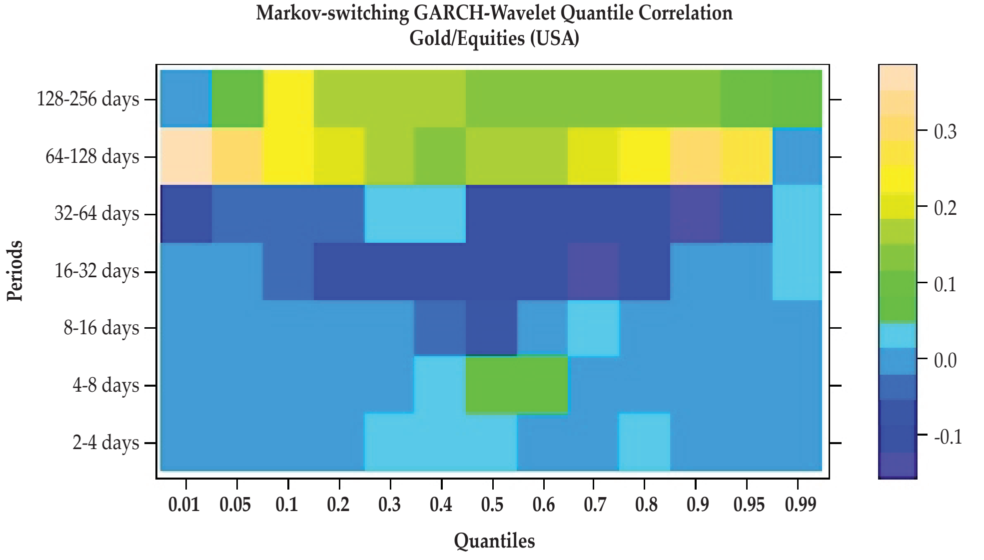


Figure 4.
MSGARCH – WQC (Gold/Equities)

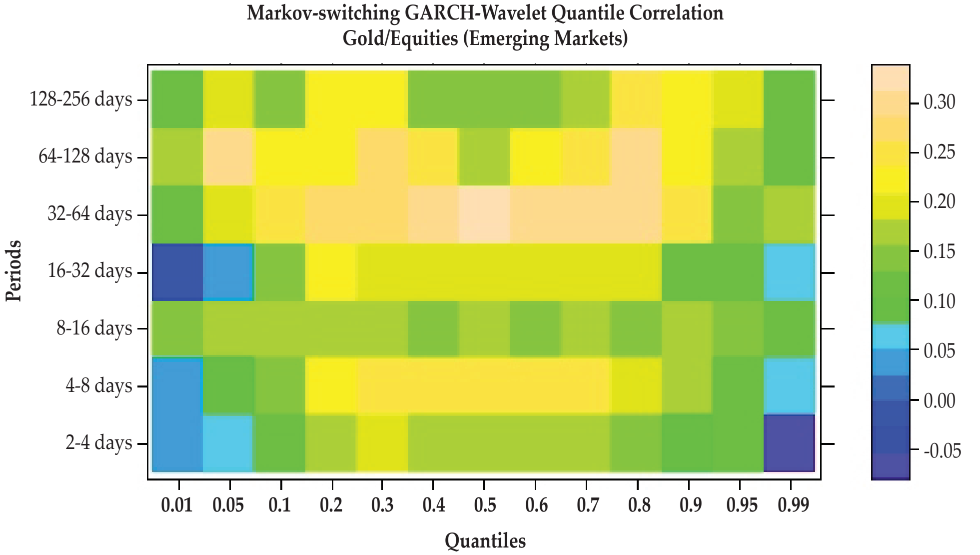


Figure 4.
MSGARCH – WQC (Gold/Equities) (Continued)

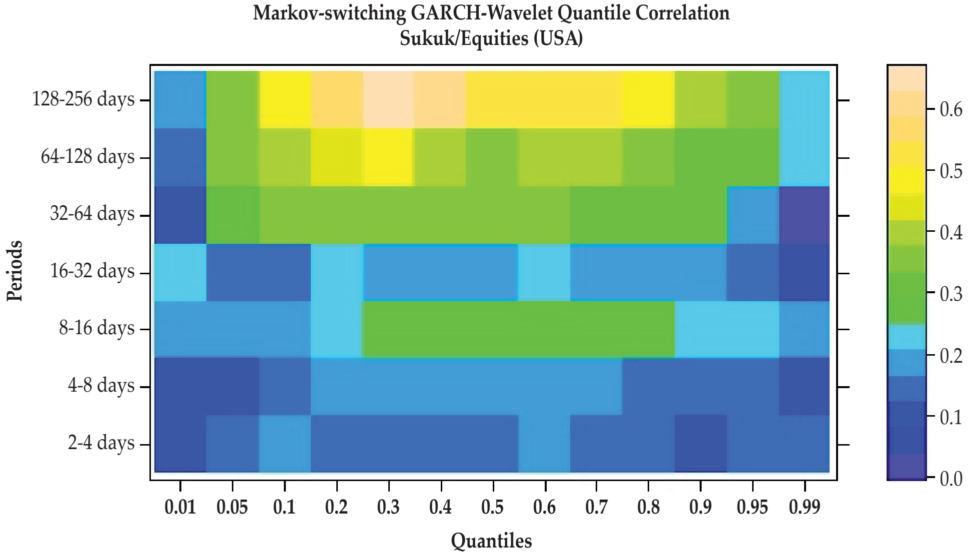


Figure 5.
MSGARCH – WQC (Sukuk/Equities)

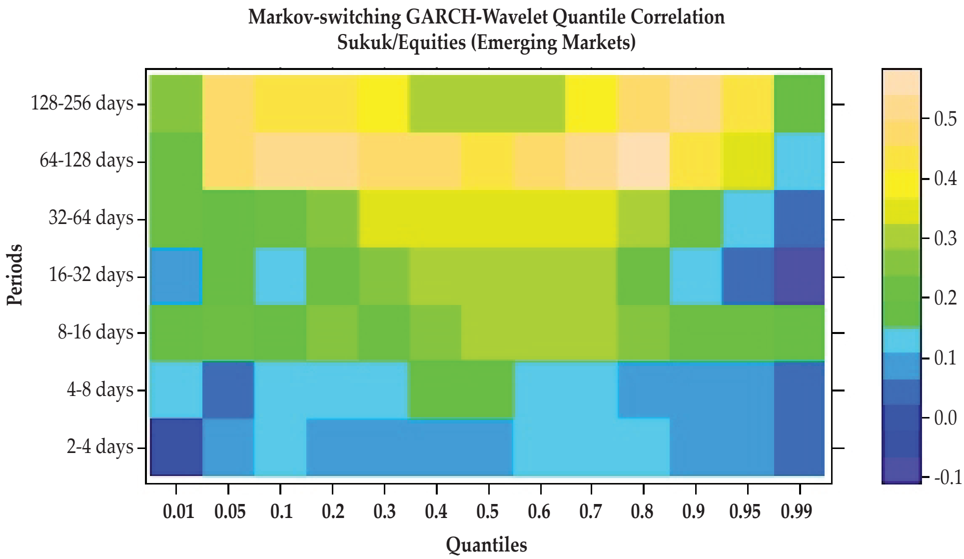


Figure 5.
MSGARCH – WQC (Sukuk/Equities) (Continued)

4.5. TVGARCH – WQC

Before estimating TVGARCH, nonstationary volatility must be detected. **Table 5** shows that the unconditional variance is not constant and has three transitions for the US equities. The results indicate that the US equities are associated with TV(3) – GARCH(1,1). Moreover, **Table 6** presents the results from TV(3) – GARCH(1,1) of US equities.

Table 5.
Testing GARCH (1.1) against TV – GARCH (1,1) in the USA

Panel A – US Equities			
Results from GARCH (1,1):			
	intercept.h	**arch1**	**garch1**
Estimate	0.0281	0.1095	0.8773
Std. Error	0.0210	0.0388	0.0471
Log-Likelihood	-1580.284		
Results from the Robust Test:			
		TR²	***p*-value**
H_0: $\vartheta_0^* = \vartheta_1^* = \vartheta_2^* = \vartheta_3^* = 0$		18.7281	0.0003
H_{03}: $\vartheta_3^* = 0$		8.8476	0.0029
H_{02}: $\vartheta_2^* = 0 \mid \vartheta_3^* = 0$		0.6781	0.4102
H_{01}: $\vartheta_1^* = 0 \mid \vartheta_2^* = \vartheta_3^* = 0$		6.9799	0.0082
No. of locations (α=0.05) = 3			

Table 5.
Testing GARCH (1.1) against TV – GARCH (1,1) in the USA (Continued)

Panel B – Gold			
Results from for GARCH (1,1):			
	intercept.h	**arch1**	**garch1**
Estimate	0.0979	0.1110	0.7887
Std. Error	0.0266	0.0321	0.0398
Log-Likelihood	-1372.765		
Results from the Robust Test:			
		TR^2	***p*-value**
H_0: $\vartheta_0^* = \vartheta_1^* = \vartheta_2^* = \vartheta_3^* = 0$		6.5805	0.0865
H_{03}: $\vartheta_3^* = 0$		0.5503	0.4582
H_{02}: $\vartheta_2^* = 0 \mid \vartheta_3^* = 0$		0.3260	0.5680
H_{01}: $\vartheta_1^* = 0 \mid \vartheta_2^* = \vartheta_3^* = 0$		5.7588	0.0164
No. of locations (α=0.05) = 0			
Panel C – Sukuk			
Results from GARCH (1,1):			
	intercept.h	**arch1**	**garch1**
Estimate	0.0028	0.1523	0.8368
Std. Error	0.0014	0.0566	0.0558
Log-Likelihood	-231.6856		
Results from the Robust Test:			
		TR^2	***p*-value**
H_0: $\vartheta_0^* = \vartheta_1^* = \vartheta_2^* = \vartheta_3^* = 0$		5.5215	0.1374
H_{03}: $\vartheta_3^* = 0$		0.3167	0.5736
H_{02}: $\vartheta_2^* = 0 \mid \vartheta_3^* = 0$		2.4728	0.1158
H_{01}: $\vartheta_1^* = 0 \mid \vartheta_2^* = \vartheta_3^* = 0$		3.2535	0.0713
No. of locations (α=0.05) = 0			

Notes: This table shows testing H_0 ($\vartheta_0^* = \vartheta_1^* = \vartheta_2^* = \vartheta_3^* = 0$), which is the unconditional variance is time-invariant for equities (Panel A), Gold (Panel B), and Sukuk (Panel C) in the US. The results only reject Panel A's null hypothesis (p-value is 0.0003).

Table 6.
The estimation results of TV (3) – GARCH (1,1) – US Equities

Long–term parameter (time-varying specification):					
	Size1	**Speed1**	**Location1**	**Location2**	**Location3**
Estimate	11.1381	5.5214	0.0940	0.0960	0.9990
Std. Error	4.3450	0.3293	0.0162	0.0163	0.0047
Short–term parameter (GARCH specification):					
			intercept.h	**arch1**	**garch1**
Estimate			0.0268	0.1246	0.8482
Std. Error			0.0105	0.0230	0.0252
Log-Likelihood			-1576.77		

Notes: This table reveals the TV (3) – GARCH (1,1) parameters of US Equities.

Similarly, **Table 7** indicates that the unconditional variance is not constant and has three transitions for the emerging market equities. Thus, the results establish that the emerging market equities are also characterized by TV(3) – GARCH(1,1). **Table 8** presents the results from TV(3) – GARCH(1,1) of Islamic equities in emerging markets.

Table 7.
Testing GARCH (1.1) against TV – GARCH (1,1) in Emerging Markets

Panel A – Equities in Emerging Markets			
Results from GARCH (1,1):			
	intercept.h	**arch1**	**garch1**
Estimate	0.0765	0.0910	0.8641
Std. Error	0.0394	0.0414	0.0521
Log-Likelihood	-1603.191		
Results from the Robust Test:			
		TR^2	***p*-value**
H_0: $\vartheta_0^* = \vartheta_1^* = \vartheta_2^* = \vartheta_3^* = 0$		9.9139	0.0193
H_{03}: $\vartheta_3^* = 0$		6.3303	0.0119
H_{02}: $\vartheta_2^* = 0 \mid \vartheta_3^* = 0$		3.3440	0.0674
H_{01}: $\vartheta_1^* = 0 \mid \vartheta_2^* = \vartheta_3^* = 0$		5.0278	0.0249
No. of locations (α=0.05) = 3			
Panel B – Gold			
Results from GARCH (1,1):			
	intercept.h	**arch1**	**garch1**
Estimate	0.0969	0.1122	0.7892
Std. Error	0.0280	0.0329	0.0424
Log-Likelihood	-1344.781		
Results from the Robust Test:			
		TR^2	***p*-value**
H_0: $\vartheta_0^* = \vartheta_1^* = \vartheta_2^* = \vartheta_3^* = 0$		6.2833	0.0986
H_{03}: $\vartheta_3^* = 0$		0.4457	0.5044
H_{02}: $\vartheta_2^* = 0 \mid \vartheta_3^* = 0$		0.3881	0.5333
H_{01}: $\vartheta_1^* = 0 \mid \vartheta_2^* = \vartheta_3^* = 0$		5.5940	0.0180
No. of locations (α=0.05) = 0			
Panel C – Sukuk			
Results from GARCH (1,1):			
	intercept.h	**arch1**	**garch1**
Estimate	0.0021	0.1188	0.8722
Std. Error	0.0008	0.0406	0.0394
Log-Likelihood	-221.7805		
Results from the Robust Test:			
		TR^2	***p*-value**
H_0: $\vartheta_0^* = \vartheta_1^* = \vartheta_2^* = \vartheta_3^* = 0$		6.8782	0.0759
H_{03}: $\vartheta_3^* = 0$		0.4922	0.4830
H_{02}: $\vartheta_2^* = 0 \mid \vartheta_3^* = 0$		2.4366	0.1185
H_{01}: $\vartheta_1^* = 0 \mid \vartheta_2^* = \vartheta_3^* = 0$		4.9087	0.0267
No. of locations (α=0.05) = 0			

Notes: This table shows testing H0 ($\vartheta_0^* = \vartheta_1^* = \vartheta_2^* = \vartheta_3^* = 0$), which is the unconditional variance is time-invariant for US equities, Gold, and Sukuk in the US. The results only reject Panel A's null hypothesis (p-value is 0.0193).

Table 8.
The estimation results of TV (3) – GARCH (1,1) – Equities in Emerging Markets

Long–term parameter (time-varying specification):					
	Size1	**Speed1**	**Location1**	**Location2**	**Location3**
Estimate	12.1501	5.5214	0.0020	0.0492	0.9990
Std. Error	9.1834	1.2106	0.1418	0.0361	0.0063
Short–term parameter (GARCH specification):					
			intercept.h	**arch1**	**garch1**
Estimate			0.1260	0.0733	0.8013
Std. Error			0.0393	0.0210	0.0434
Log-Likelihood			-1595.148		

Notes: This table reveals the TV (3) – GARCH (1,1) parameters of US Equities.

Figure 6 reveals the conditional standard deviation for both a stationary GARCH(1,1) and the TV(3) – GARCH(1,1) for US equities. The figures show that the stationary GARCH (1,1) understates the magnitude of the conditional standard deviation, especially during COVID–19, Russia – Ukraine, and the Middle Eastern crises.

Similarly, **Figure 7** exhibits the conditional standard deviation for both a stationary GARCH(1,1) and the TV(3) – GARCH(1,1) for emerging market equities. The figures illustrate that the stationary GARCH(1,1) also understates the magnitude of the conditional standard deviation in emerging market equities.

Similar to the results from MSGARCH-WQC, **Figures 8** and **9** reveal that gold and Sukuk are haven assets. However, MSGARCH-WQC indicates that Sukuk is not a haven asset for US equities, while TVGARCH-WQC reveals that Sukuk is a haven asset for US equities (see 0.1 quintile of 128 – 256 days on **Figure 9**). The results from TVGARCH are more robust since the TVGARCH package (the *R* statistical program) has more features than MSGARCH, such as nonstationary testing, higher asymmetry order, and smooth transitions.

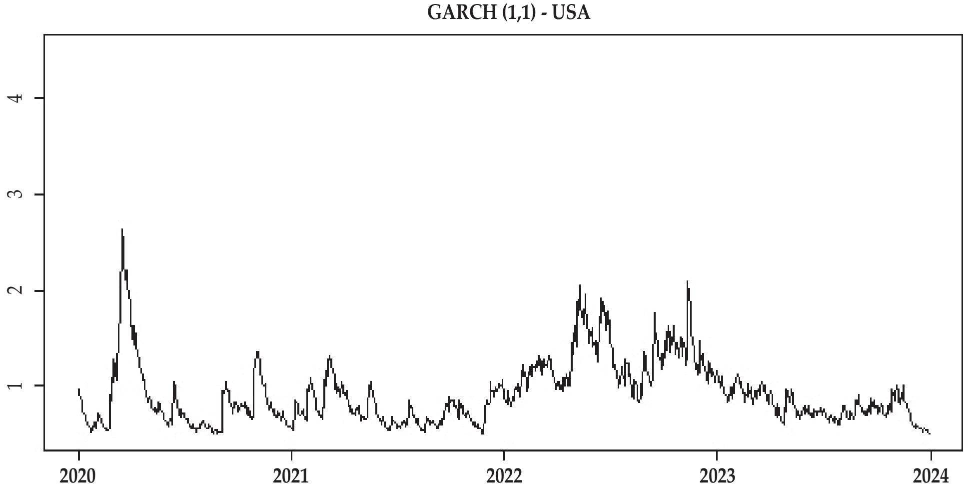


Figure 6.
Volatility based on GARCH(1,1) and TV(3) – GARCH(1,1) of US Equities

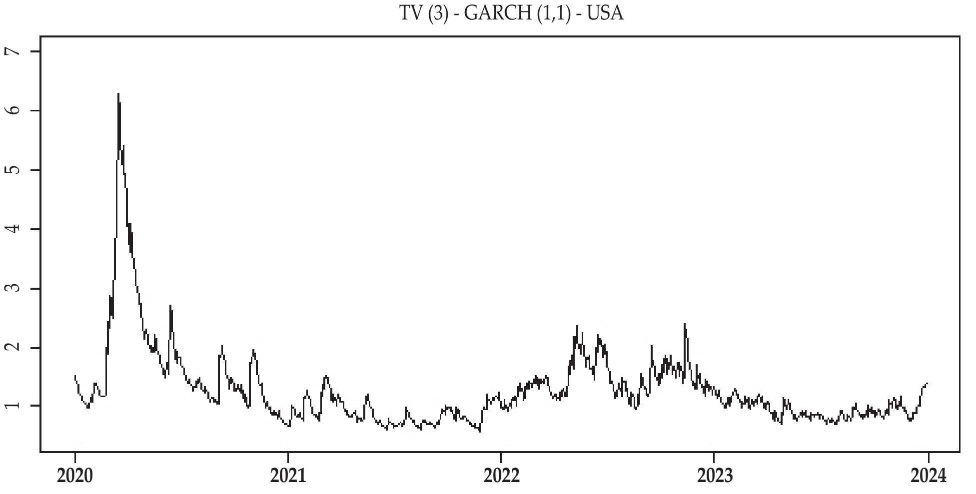


Figure 6.
Volatility based on GARCH(1,1) and TV(3) – GARCH(1,1) of US Equities (Continued)

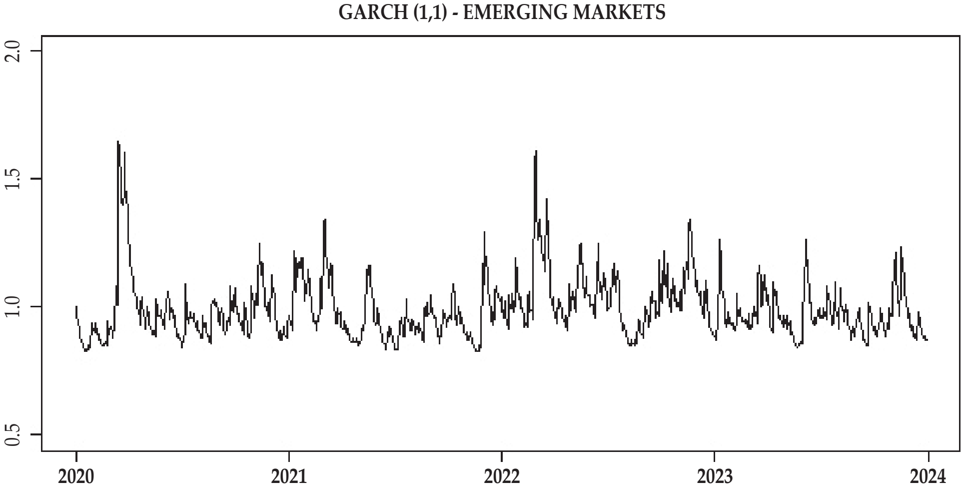


Figure 7.
Volatility based on GARCH(1,1) and TV(3) – GARCH(1,1) of Equities in Emerging Markets

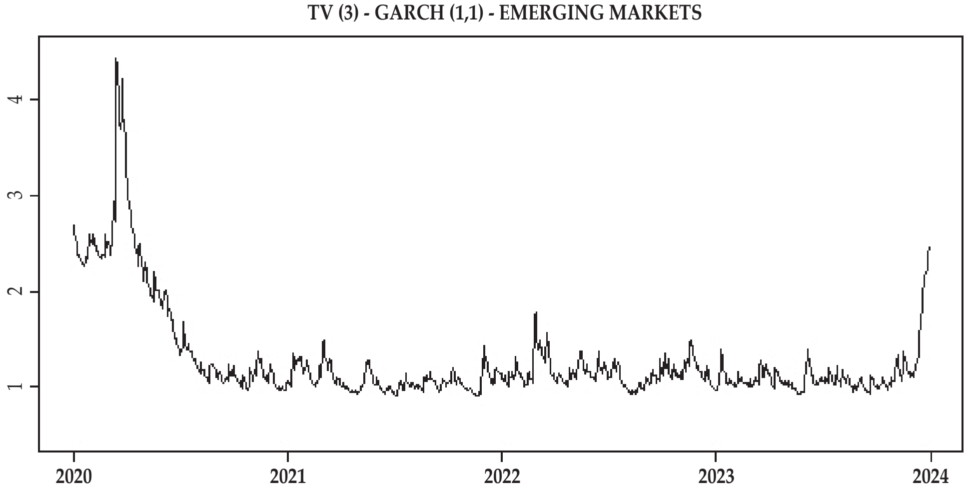


Figure 7.
Volatility based on GARCH(1,1) and TV(3) – GARCH(1,1) of Equities in Emerging Markets (Continued)

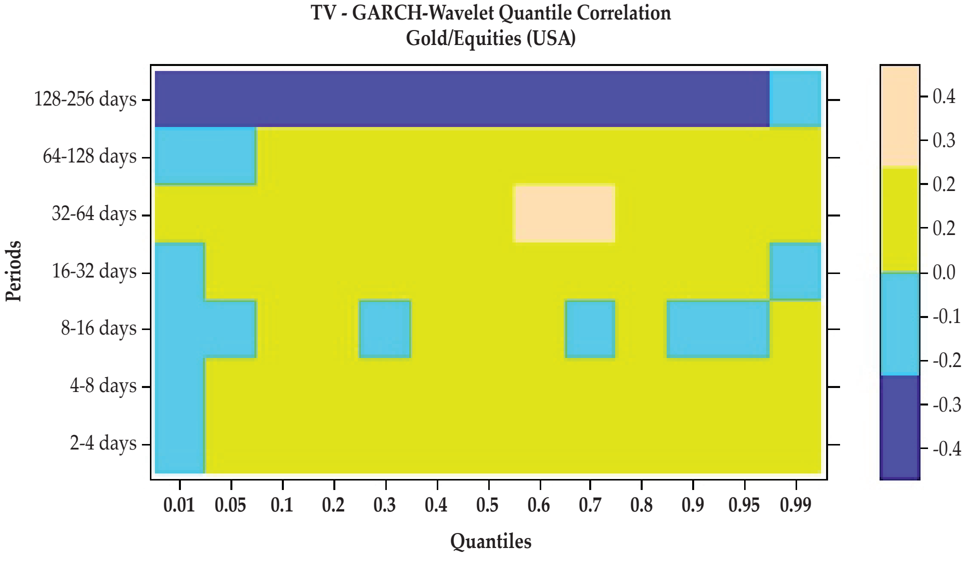


Figure 8.
TVGARCH – WQC (Gold/Equities)

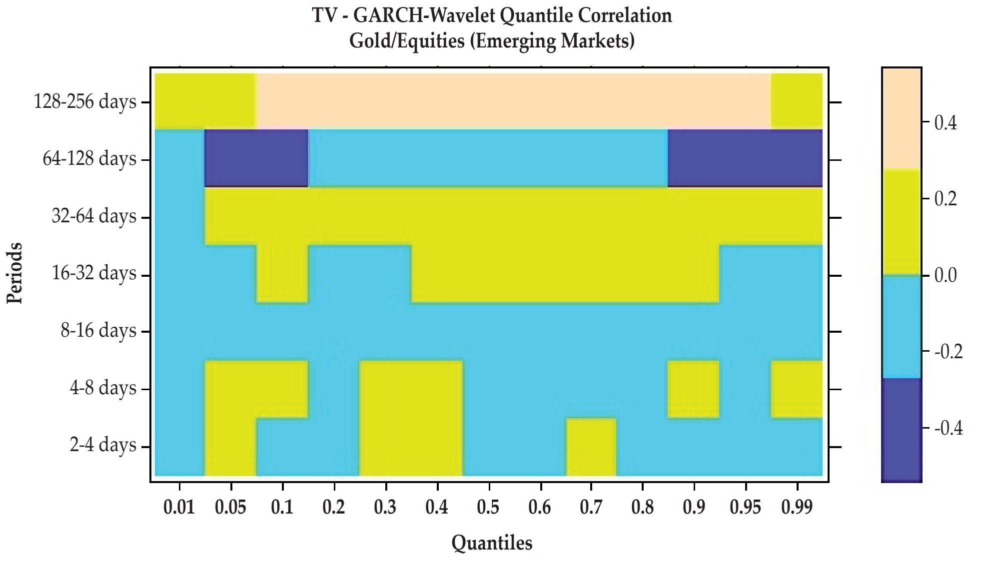


Figure 8.
TVGARCH – WQC (Gold/Equities) (Continued)

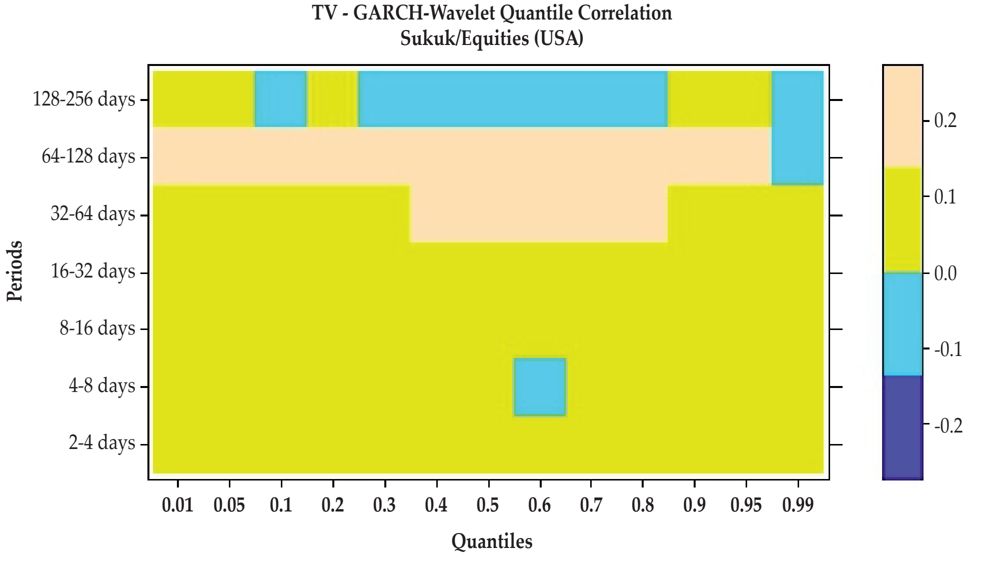


Figure 9.
TVGARCH – WQC (Sukuk/Equities)

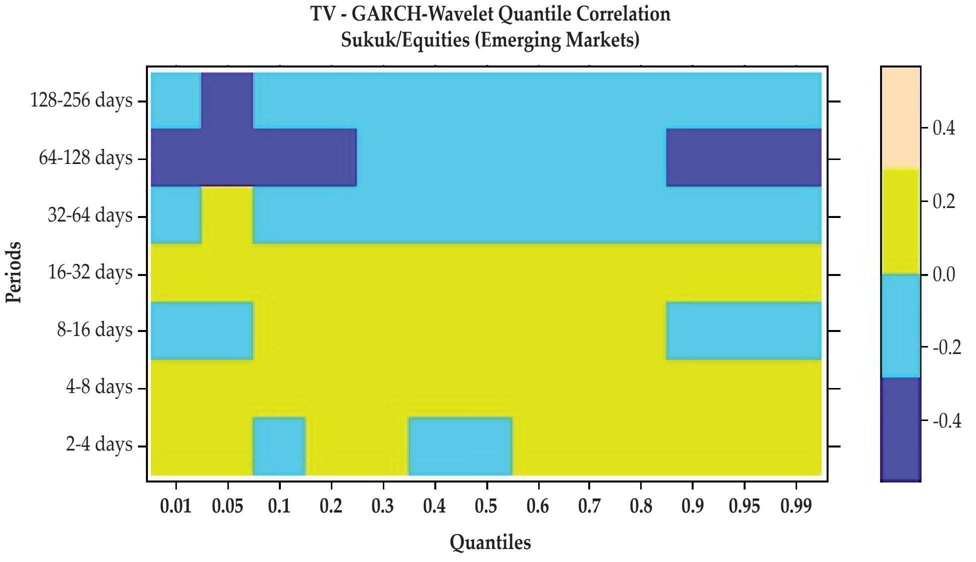


Figure 9.
TVGARCH – WQC (Sukuk/Equities) (Continued)

4.6. Simulation of TV – GARCH

Sukuk ETF is a relatively new instrument in financial markets. Hence, it is expected that the assets under the management of the Sukuk ETF will be greatly lower than those of the gold ETF. This research simulates 10000 real equities, gold, and Sukuk returns to minimize size bias.

Figures 10 and **11** exhibit the simulated returns of the haven assets and Islamic equities in the US and emerging markets, respectively. TV(3) – GARCH(1,1) parameters are used to simulate the inflation-adjusted returns of equities, while GARCH(1,1) parameters are applied to simulate the inflation-adjusted returns of the haven assets. Simply put, the parameters used for the simulation are from **Tables 6** and **8** for Islamic equities in the US and emerging markets, respectively, and **Tables 5** and **7** for the haven assets (gold and Sukuk) in the US and emerging markets, respectively.

Based on the simulated returns, this research re-applies the new approach proposed in this study (a combination of TVARCH and WQC). **Figures 12** and **13** show the results. Overall, the results reveal that gold and Sukuk are haven assets.

4.7. Portfolio Performance

The dual momentum strategy has been explained in **Section 3.** In the first part of this section, We discuss the determination of the default weights of the strategy. **Figure 14** depicts the downside volatility and Sortino ratio throughout a wide range of Sukuk/Gold – Islamic equities allocations. When combining 45% Gold or Sukuk (whichever has the best performance in the last three months) with 55% Islamic equities, the downside volatility is 0.334. As expected, the downside volatility of this proportion is smaller than that of the Islamic equities (see **Table 2**) due to the safe-haven property of gold. However, the reduction of the downside volatility comes at the cost of the Sortino ratio. Thus, we use the following allocations for the dual momentum strategy: 55% Islamic equities – 45% Sukuk or Gold (whichever has the best performance in the last three months).

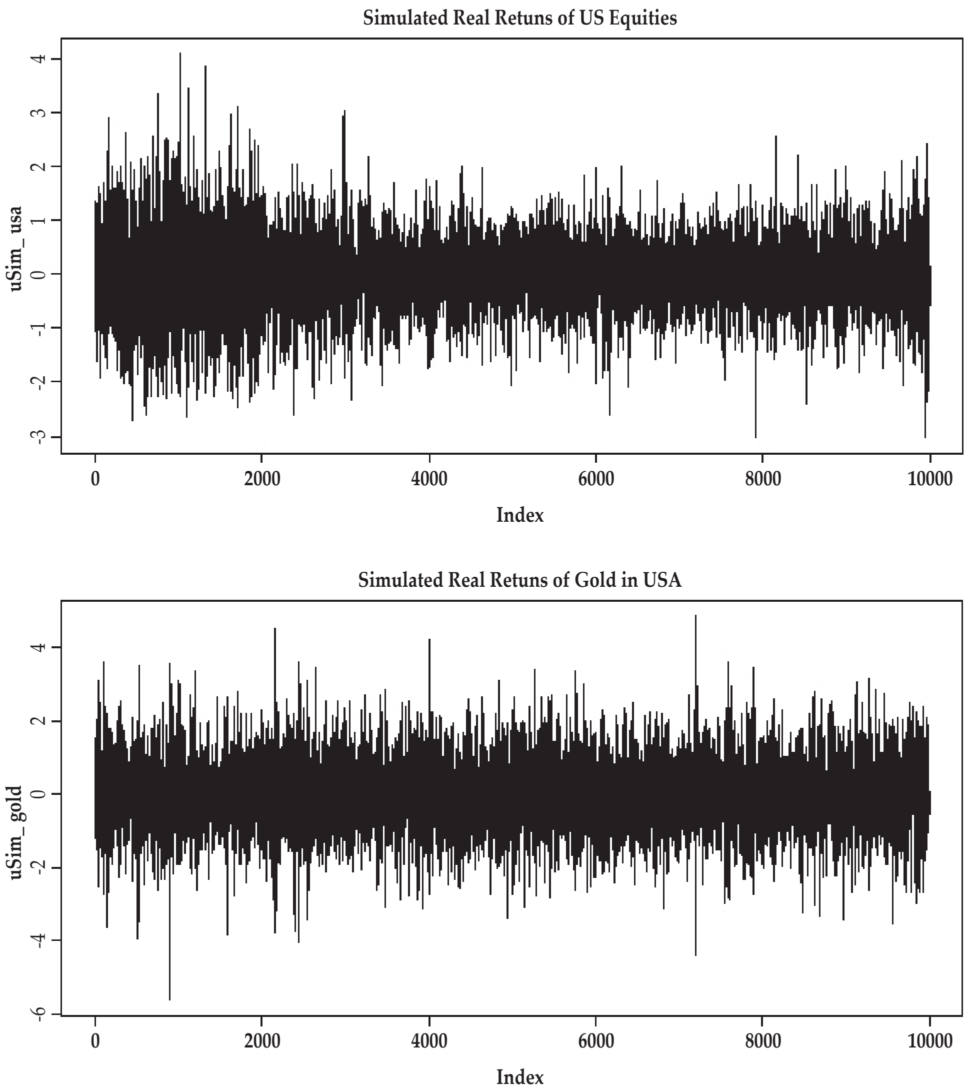


Figure 10.
Simulated Real Returns in the USA

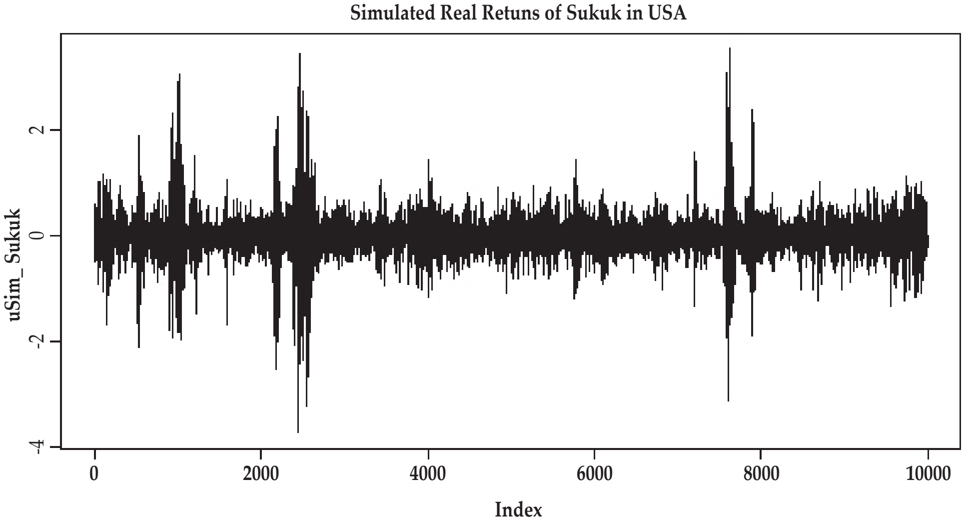


Figure 10.
Simulated Real Returns in the USA (Continued)

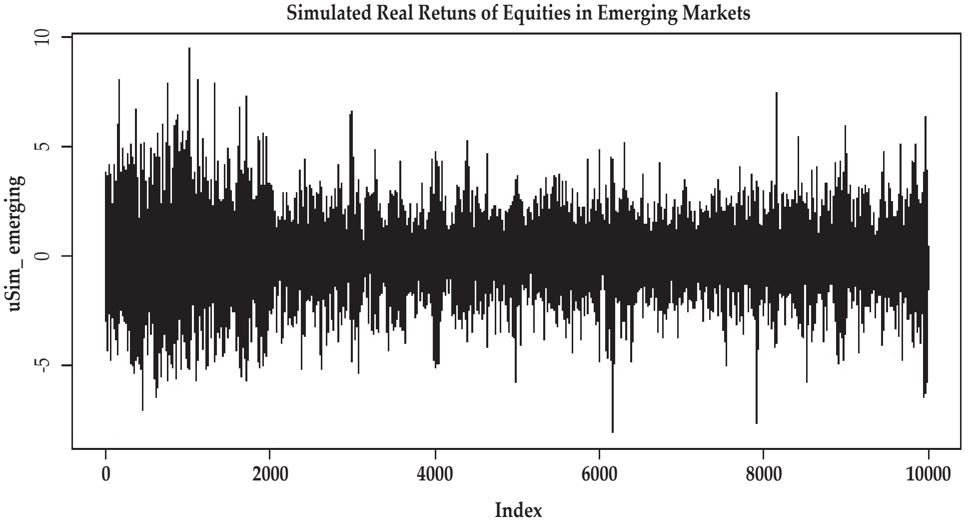


Figure 11.
Simulated Real Returns in Emerging Markets

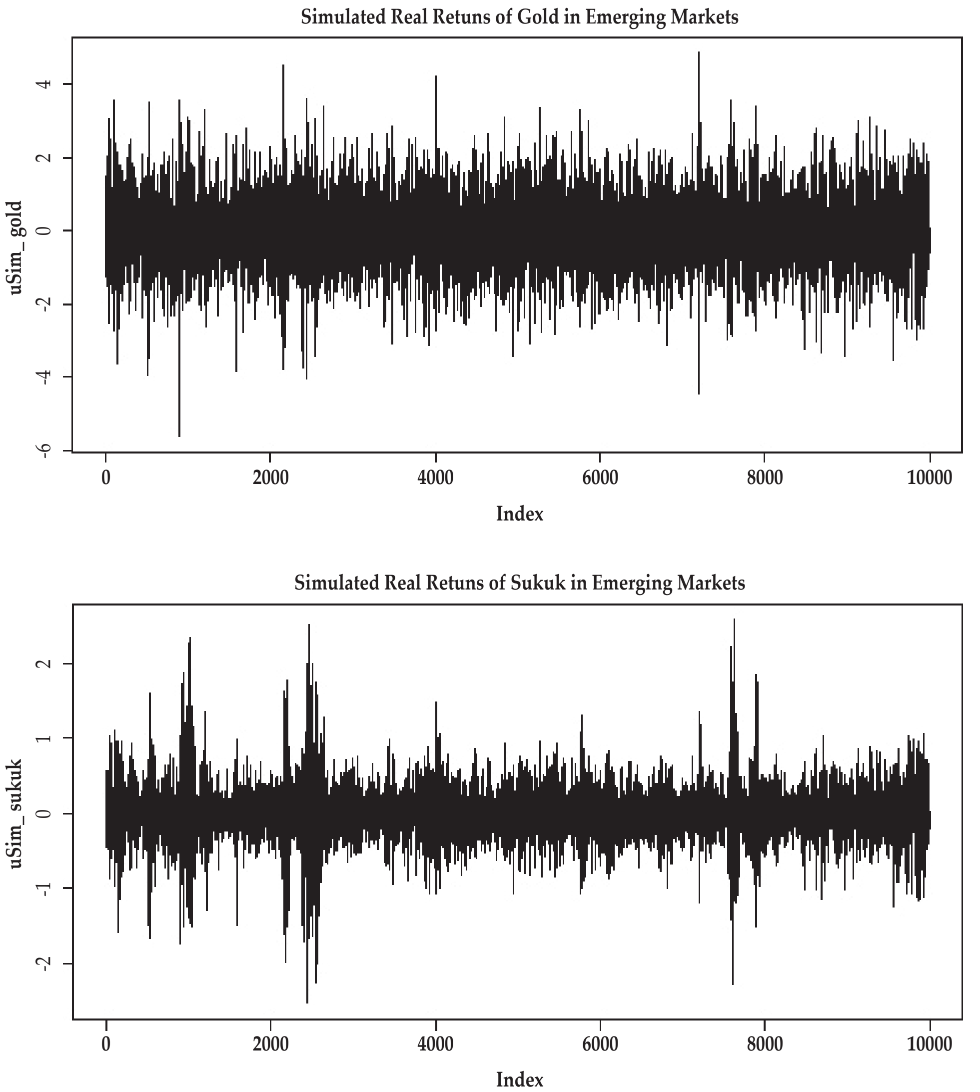


Figure 11.
Simulated Real Returns in Emerging Markets (Continued)

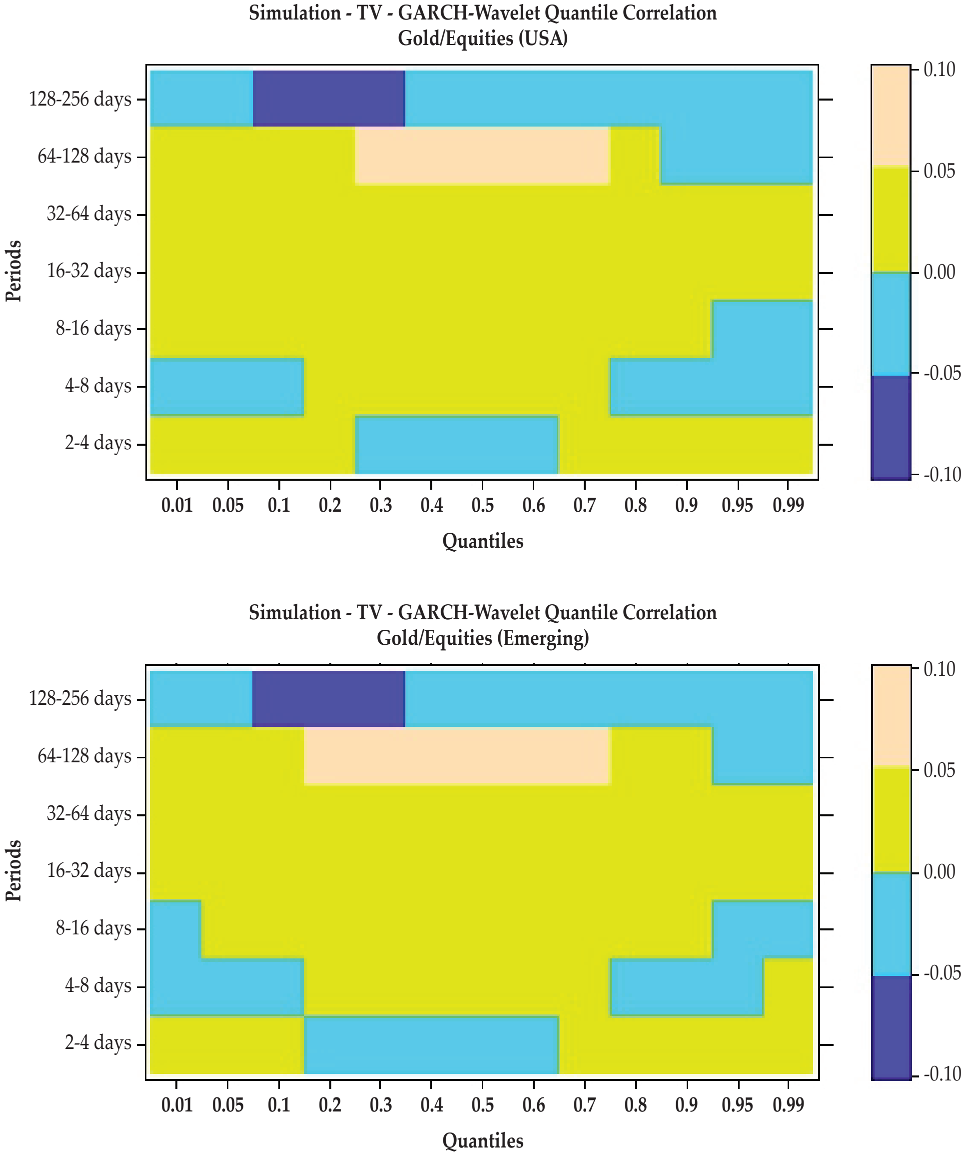
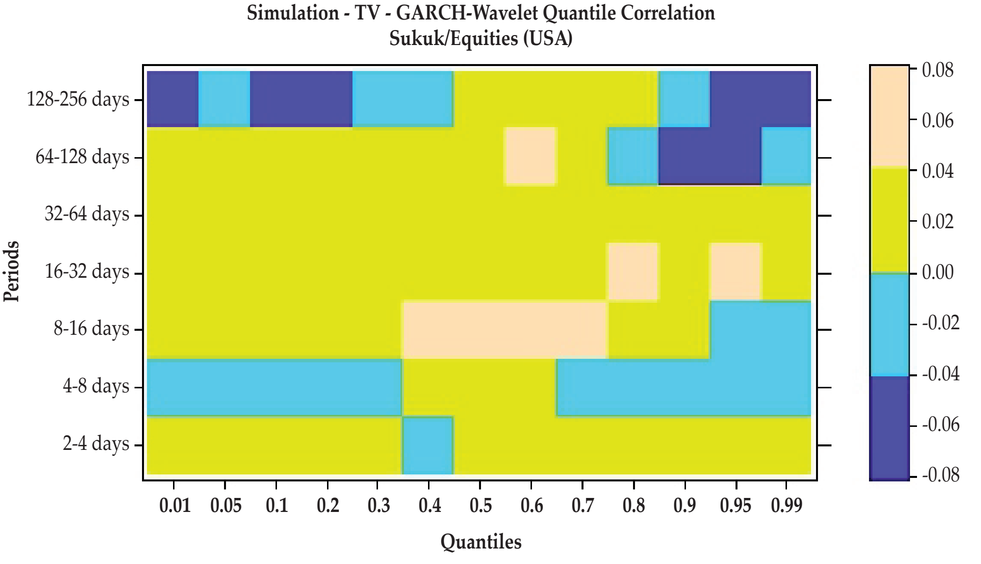


Figure 12.
Simulation – TVGARCH – WQC (Gold/Equities)



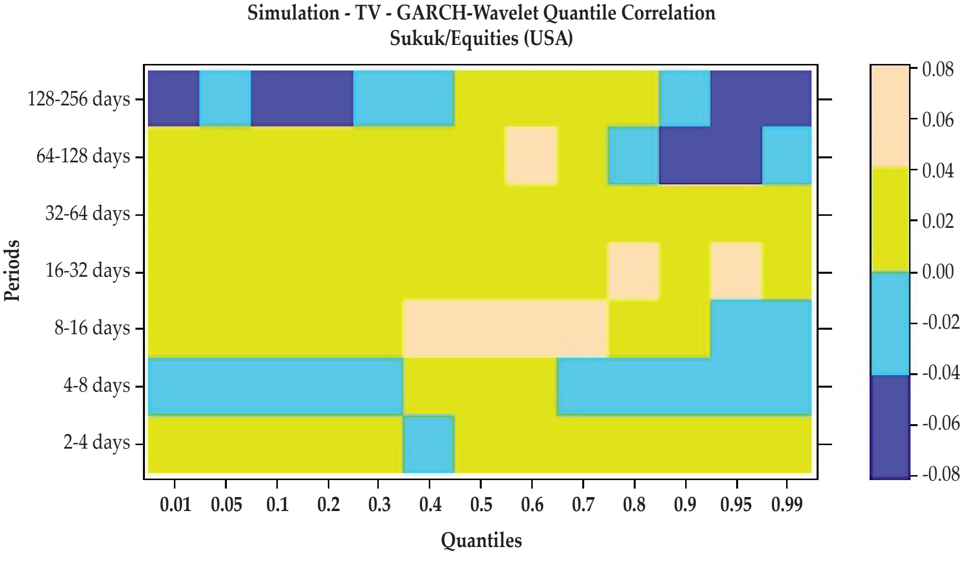


Figure 13.
Simulation – TVGARCH – WQC (Sukuk/Equities)

4.7.1. Portfolio Performance (USA)

Figure 15 depicts the portfolio weights. The rebalancing period is quarterly instead of monthly to minimize turnover. Yearly rebalancing is more economical but does not quickly adapt to market dynamics. In addition, short-selling is not allowed since it is forbidden from an Islamic point of view. The minimum variance approach allocates the majority of the assets to Sukuk. Similarly, the passive and the risk parity methods allocate a large proportion of the portfolio to Sukuk. Interestingly, the dual momentum approach indicates that investors should convert some portfolio assets to cash to avoid large drawdowns.

Further, **Table 9** shows that the dual momentum strategy outperforms the benchmark (naïve) approach regarding risk-adjusted returns (Sharpe, Sortino, and Omega). Additionally, the minimum variance strategy has the best Value-at-Risk at the cost of risk-adjusted returns. Interestingly, the dual momentum strategy has the best drawdowns. As expected, the all-equities portfolio has the worst drawdowns.

Table 9.
Portfolio Performance (the USA)

	Naive	Dual Momentum	Min. Variance	Risk Parity	All Equities	Passive
Mean returns %	-2.091	-1.173	-7.235	-5.069	5.130	-3.797
Std. deviation %	8.860	7.422	4.942	6.130	18.091	7.054
Downside Volatility %	0.397	0.330	0.209	0.268	0.820	0.315
Minimum returns %	-2.530	-1.803	-1.541	-1.753	-4.669	-1.769
Maximum returns %	2.951	1.872	1.958	1.721	5.478	1.904
Skewness	-0.033	0.016	0.529	0.169	-0.160	0.011
Kurtosis	1.836	2.004	4.306	1.614	1.575	1.531
Sharpe %	-0.912	-0.615	-6.173	-3.226	1.090	-2.063
Sortino %	-2.067	-1.400	-12.789	-7.203	2.516	-4.670
Omega	0.962	0.970	0.773	0.872	1.049	0.914
Value-at-Risk %	-0.910	-0.756	-0.465	-0.624	-1.868	-0.730
Median drawdown %	4.377	3.181	23.970	21.776	6.159	4.162
Maximum drawdown %	20.522	16.681	23.970	21.776	29.509	20.331

Notes: This table shows the portfolio performance. A positive skewness (favorable) typically has minor losses and few significant gains. The larger the risk-adjusted returns (Sharpe, Sortino, and Omega), the better the performance. The larger the drawdown, the more substantial the decrease in an investment's peak-to-trough over time and the riskier the investment. Similarly, the larger the downside volatility, measured by semi-deviation, which assesses the below mean fluctuations, the riskier the investment. In addition, the lower the value-at-risk, a loss that we are confident will not be surpassed if the portfolio is held for a certain period, the riskier the investment.

Emerging Markets																					
Equities (%)	100	95	90	85	80	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5	0
Gold or Sukuk (%)	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
Downside volatility (%)	0.425	0.407	0.391	0.376	0.363	0.352	0.344	0.338	0.335	**0.334**	0.336	0.341	0.348	0.357	0.369	0.382	0.397	0.414	0.432	0.451	0.472
Sortino ratio (%)	-5.714	-6.089	-6.470	-6.849	-7.215	-7.558	-7.866	-8.126	-8.329	-8.469	-8.543	-8.551	-8.500	-8.396	-8.249	-8.068	-7.861	-7.637	-7.404	-7.168	-6.932
USA																					
Equities (%)	100	95	90	85	80	75	70	65	60	55	50	45	40	35	30	25	20	15	10	5	0
Gold or Sukuk (%)	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
Downside volatility (%)	0.442	0.419	0.398	0.379	0.363	0.349	0.337	0.329	0.324	**0.323**	0.325	0.330	0.339	0.351	0.366	0.383	0.403	0.424	0.447	0.472	0.498
Sortino ratio (%)	-4.817	-4.727	-4.604	-4.442	-4.236	-3.979	-3.665	-3.295	-2.873	-2.407	-1.911	-1.400	-0.893	-0.402	0.062	0.491	0.882	1.237	1.555	1.839	2.092

Figure 14.
The Determination of the Default Weights of the Dual Momentum Strategy

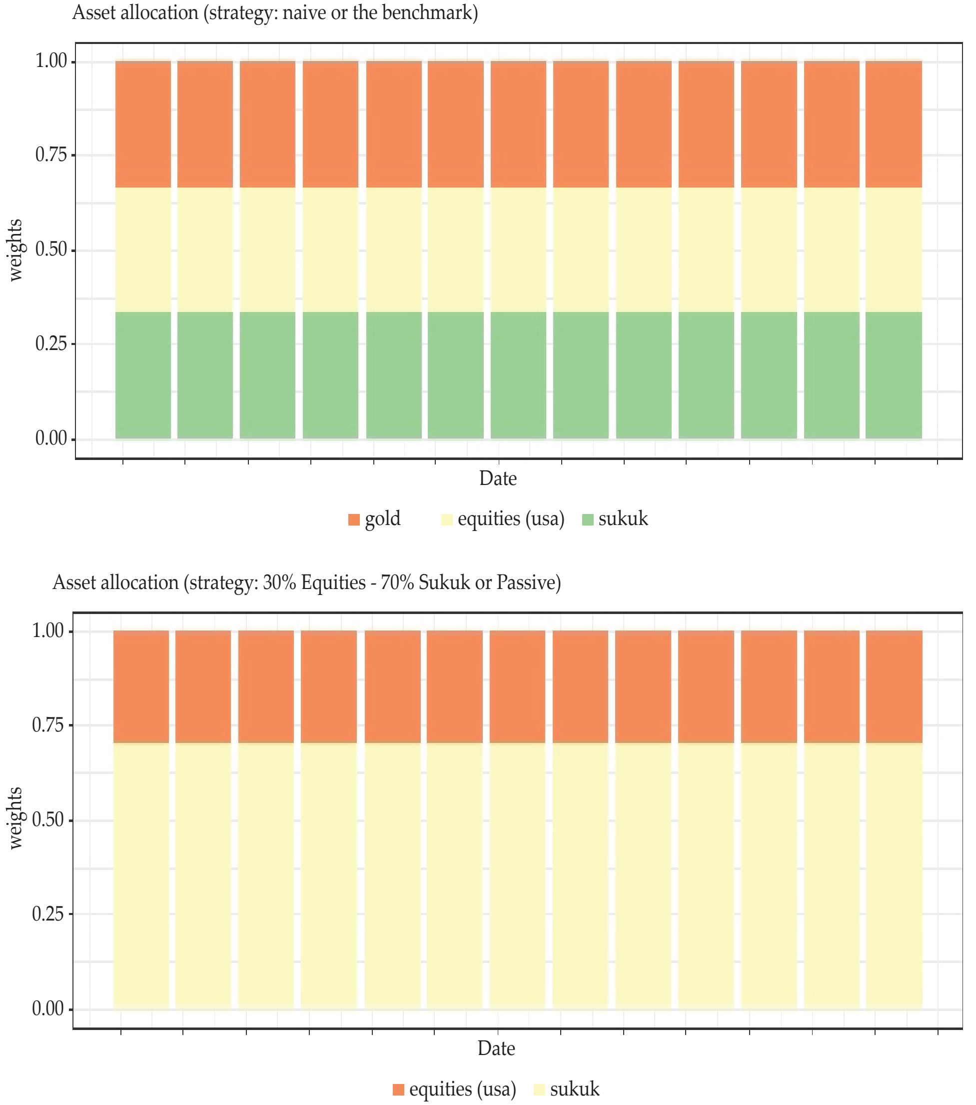


Figure 15.
Portfolio Weights (the USA)

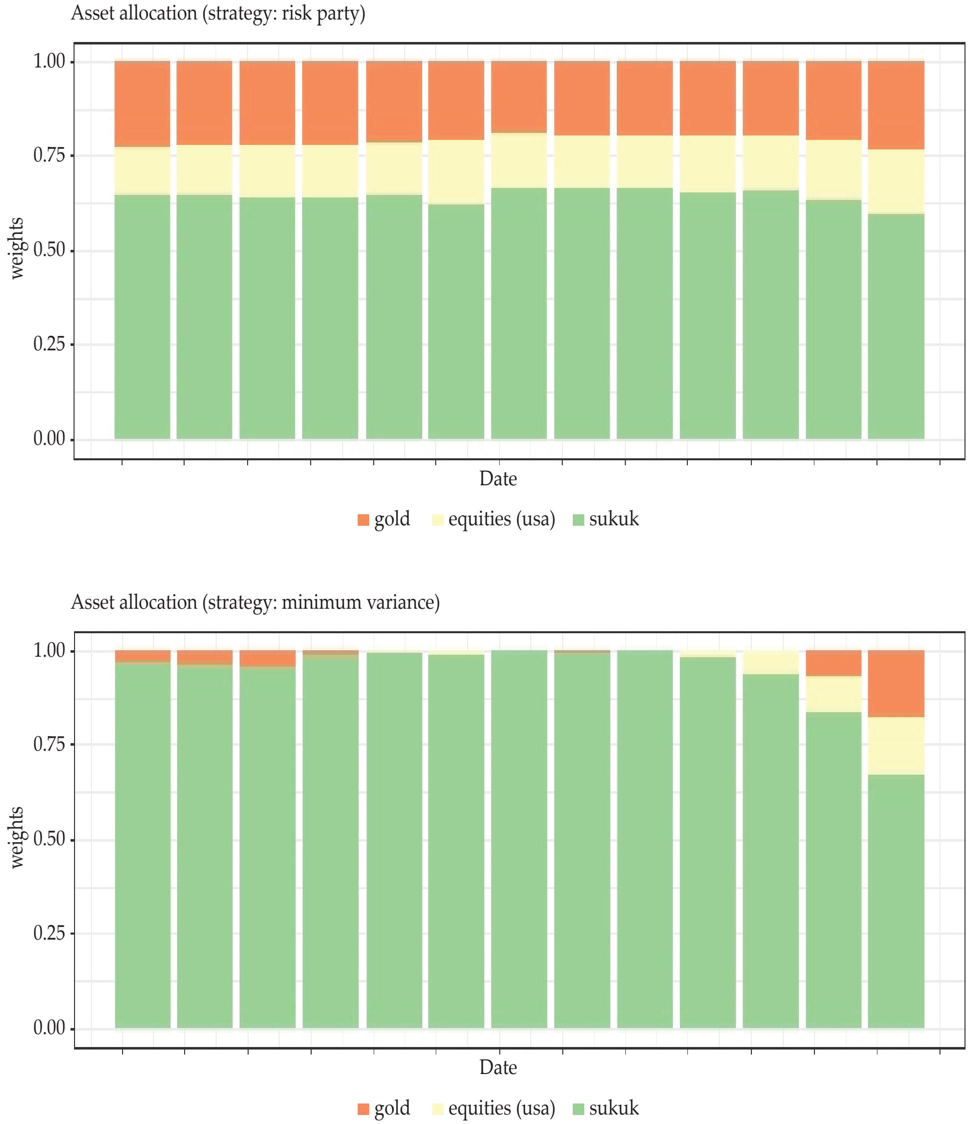


Figure 15.
Portfolio Weights (the USA) (Continued)

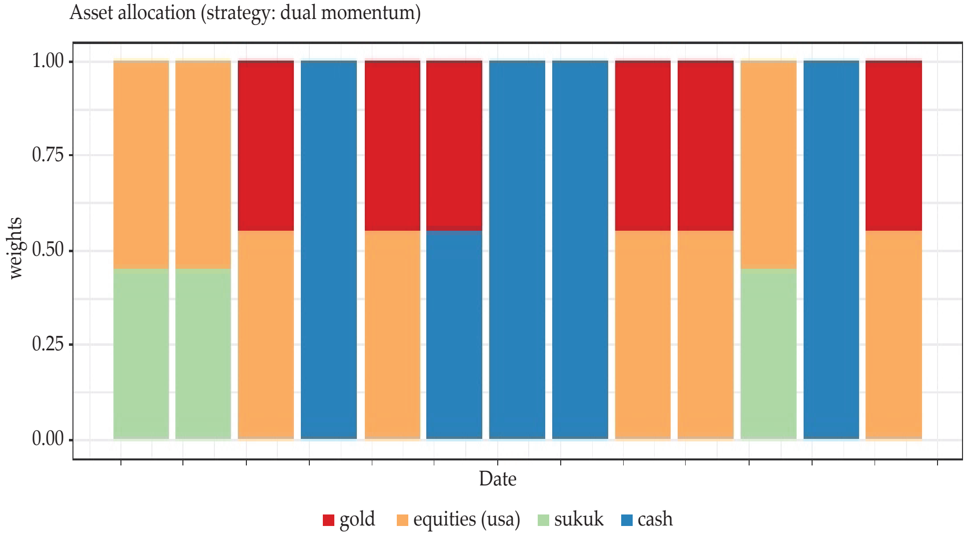


Figure 15.
Portfolio Weights (the USA) (Continued)

4.7.2. Portfolio Performance (Emerging Markets)

Figure 16 depicts the portfolio weights for emerging markets. The rebalancing period is quarterly. Also, short-selling is not allowed. The minimum variance approach allocates the majority of the time to Sukuk. Compared to the dual momentum approach in the USA, the dual momentum strategy in emerging markets indicates that investors should retain more cash to avoid more significant drawdowns. It is a very reasonable action since the drawdowns in Islamic equities in emerging markets are more prominent (see **Table 2**).

Moreover, **Table 10** shows that the dual momentum strategy outperformes other strategies regarding risk-adjusted returns (Sharpe, Sortino, and Omega). Additionally, the minimum variance strategy has the best Value-at-Risk. Similarly, the 70% Sukuk – 30% Islamic equities portfolio has the second-best Value-at-Risk. As predicted, the all-equities portfolio has the worst drawdowns. At the same time, the dual momentum strategy has the best drawdowns.

Table 10.
Portfolio Performance (Emerging Markets)

	Naive	Dual Momentum	Min. Variance	Risk Parity	All Equities	Passive
Mean returns %	-10.042	-7.938	-8.814	-9.202	-15.549	-10.901
Std. deviation %	9.041	8.473	4.954	6.084	17.784	6.831
Downside Volatility %	0.399	0.368	0.206	0.26	0.787	0.295
Minimum returns %	-2.087	-2.235	-1.189	-1.504	-5.241	-1.702
Maximum returns %	2.114	2.585	1.943	1.707	3.712	1.829
Skewness	0.039	0.275	0.697	0.34	-0.017	0.256
Kurtosis	0.564	4.033	4.010	1.302	0.751	1.401
Sharpe %	-4.137	-3.828	-7.637	-5.904	-3.262	-6.123
Sortino %	-9.442	-8.249	-15.479	-12.963	-7.511	-13.514
Omega	0.836	0.852	0.733	0.782	0.867	0.770
Value-at-Risk %	-0.963	-0.823	-0.458	-0.619	-1.892	-0.706
Median drawdown %	17.178	7.122	26.592	28.475	13.293	8.775
Maximum drawdown %	32.905	26.246	26.592	28.475	50.283	33.759

Notes: This table shows the portfolio performance. A positive skewness (favorable) typically has minor losses and few significant gains. The larger the risk-adjusted returns (Sharpe, Sortino, and Omega), the better the performance. The larger the drawdown, the more substantial the decrease in an investment's peak-to-trough over time and the riskier the investment. Similarly, the larger the downside volatility, measured by semi-deviation, which assesses the below mean fluctuations, the riskier the investment. In addition, the lower the value-at-risk, a loss that we are pretty confident will not be surpassed if the portfolio is held for a certain period, the riskier the investment.

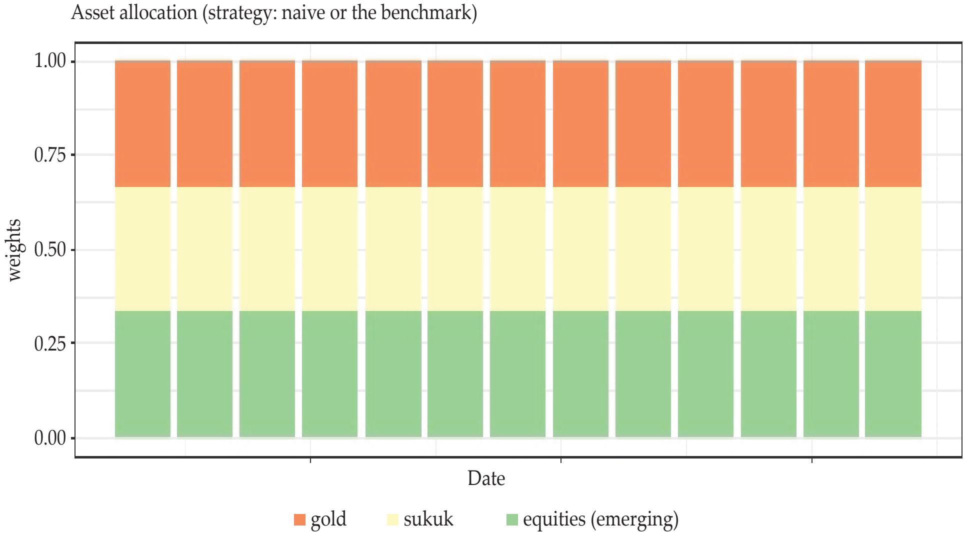
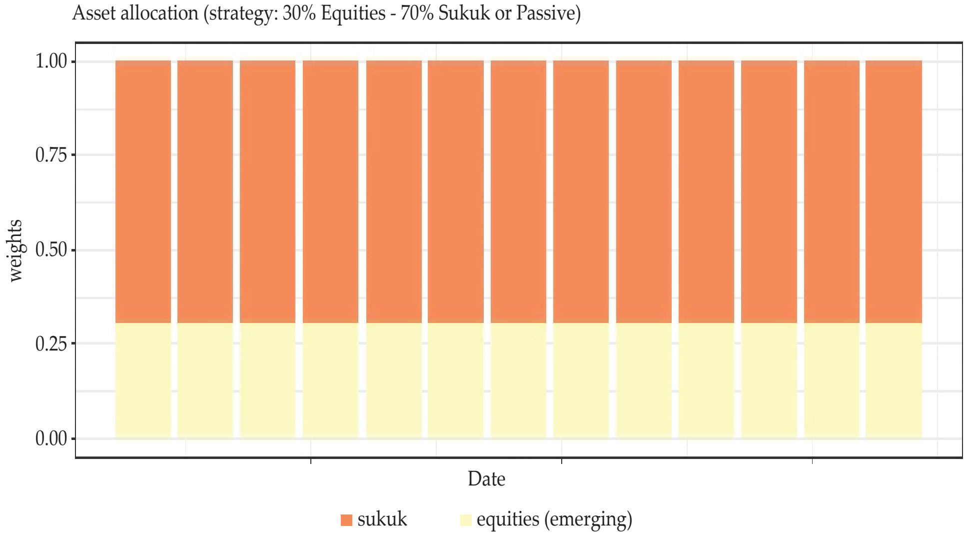


Figure 16.
Portfolio Weights (Emerging Markets)



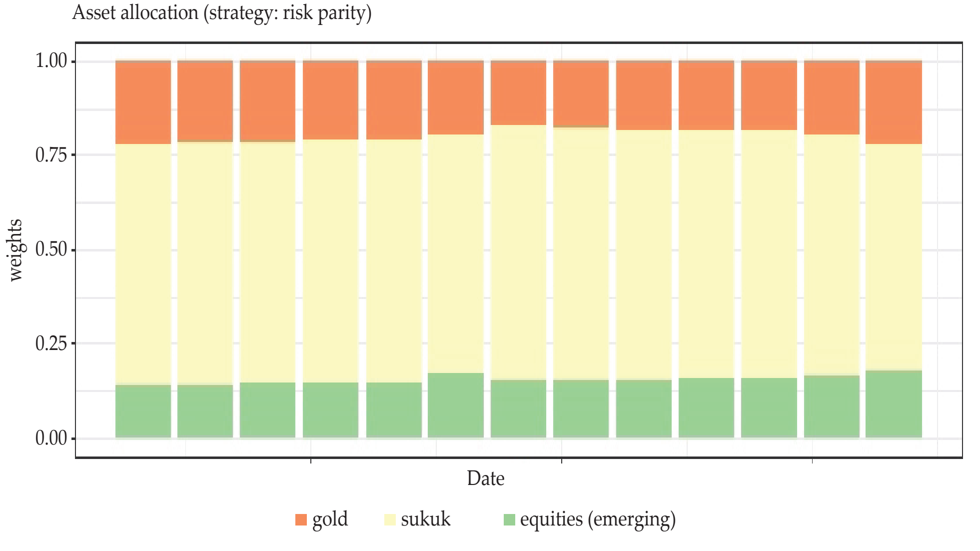
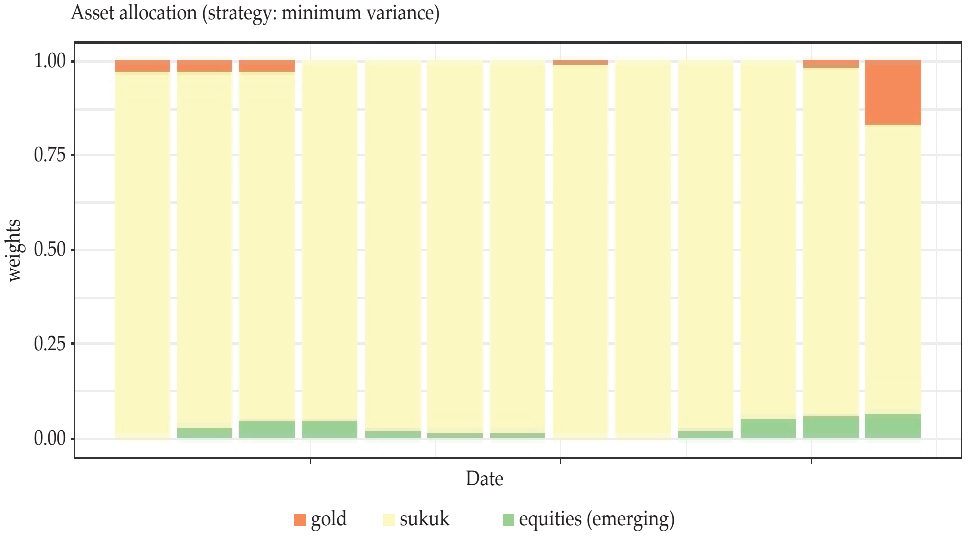


Figure 16.
Portfolio Weights (Emerging Markets) (Continued)



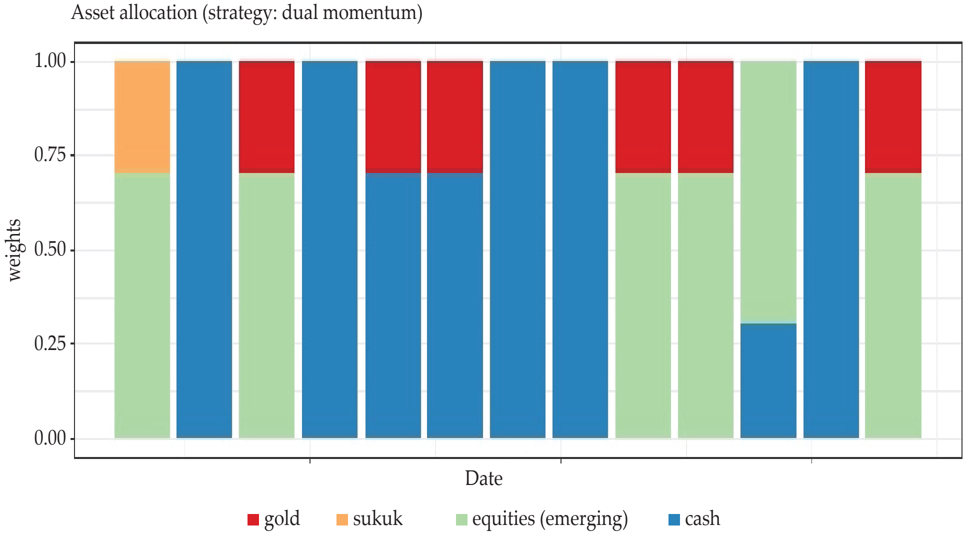


Figure 16.
Portfolio Weights (Emerging Markets) (Continued)

4.8. Robustness Tests

The results in the previous sections are based on the followings: (i) there is no transaction cost, and (ii) we use the entire data. We re-evaluate portfolio performance pertaining to these two aspects

4.8.1. Transaction Cost

We impose a transaction cost of 63 basis points (Angel et al., 2016) and then reassess the portfolio performance. **Tables 11** and **12** show that the dual momentum strategy still has the best drawdown, although the transaction cost slightly reduces the risk-adjusted returns.

Table 11.
Portfolio Performance (the USA) – Net of Transaction Cost

	Naïve	Dual Momentum	Min. Variance	Risk Parity	All Equities	Passive
Mean returns %	-2.272	-2.386	-7.307	-5.225	5.495	-3.703
Std. deviation %	8.863	7.481	4.949	6.132	18.092	7.053
Downside Volatility %	0.397	0.333	0.209	0.268	0.820	0.315
Minimum returns %	-2.530	-1.803	-1.541	-1.753	-4.669	-1.769
Maximum returns %	2.951	1.872	1.958	1.721	5.478	1.904
Skewness	-0.031	0.010	0.527	0.171	-0.162	0.009
Kurtosis	1.833	1.893	4.286	1.612	1.580	1.538
Sharpe %	-0.990	-1.234	-6.219	-3.322	1.168	-2.013
Sortino %	-2.244	-2.804	-12.887	-7.417	2.696	-4.557
Omega	0.958	0.940	0.771	0.868	1.052	0.916
Value-at-Risk %	-0.911	-0.767	-0.466	-0.624	-1.867	-0.730
Median drawdown %	4.389	3.389	24.195	21.890	6.159	4.751
Maximum drawdown %	20.607	18.755	24.195	21.890	29.509	20.395

Notes: This table shows the portfolio performance. A positive skewness (favorable) typically has minor losses and few significant gains. The larger the risk-adjusted returns (Sharpe, Sortino, and Omega), the better the performance. The larger the drawdown, the more significant the decrease in an investment's peak-to-trough over time and the riskier the investment. Similarly, the larger the downside volatility, measured by semi-deviation, which assesses the below mean fluctuations, the riskier the investment. In addition, the lower the value-at-risk, a loss that we are pretty confident will not be surpassed if the portfolio is held for a certain period, the riskier the investment.

Table 12.
Portfolio Performance (Emerging Markets) – Net of Transaction Cost

	Naïve	Dual Momentum	Min. Variance	Risk Parity	All Equities	Passive
Mean returns %	-10.433	-9.318	-8.872	-9.435	-15.824	-10.978
Std. deviation %	9.030	8.510	4.958	6.081	17.790	6.835
Downside Volatility %	0.399	0.370	0.207	0.260	0.787	0.295
Minimum returns %	-2.087	-2.235	-1.189	-1.504	-5.241	-1.702
Maximum returns %	2.114	2.585	1.943	1.707	3.712	1.829
Skewness	0.040	0.286	0.696	0.343	-0.015	0.258
Kurtosis	0.575	3.942	4.002	1.313	0.751	1.396
Sharpe %	-4.298	-4.449	-7.674	-6.050	-3.318	-6.161
Sortino %	-9.800	-9.572	-15.556	-13.273	-7.638	-13.597
Omega	0.830	0.790	0.731	0.777	0.865	0.768
Value-at-Risk %	-0.963	-0.831	-0.459	-0.619	-1.892	-0.707
Median drawdown %	17.244	7.764	28.618	28.814	13.293	8.794
Maximum drawdown %	33.038	26.771	28.618	28.814	50.283	33.834

Notes: This table shows the portfolio performance. A positive skewness (favorable) typically has minor losses and few significant gains. The larger the risk-adjusted returns (Sharpe, Sortino, and Omega), the better the performance. The larger the drawdown, the more significant the decrease in an investment's peak-to-trough over time and the riskier the investment. Similarly, the larger the downside volatility, measured by semi-deviation, which assesses the below mean fluctuations, the riskier the investment. In addition, the lower the value-at-risk, a loss that we are pretty confident will not be surpassed if the portfolio is held for a certain period, the riskier the investment.

4.8.2. New Data Partitioning

Our earlier results are based on the entire sample, including low and high downturn periods. Simply put, this exercise can help determine the long-term cost of administering a trading strategy. Therefore, we would want a plan that does not suffer significant returns reduction during regular and bull markets (by including haven assets in the equity portfolio) while reducing downside risk during substantial market drawdowns.

For robustness, this study investigates the performance of the trading strategies during large drawdowns. This research selects all periods in which equities experienced more than 10% drawdowns while maintaining a rolling window approach. **Figures 17** and **18** show the equities drawdowns throughout the years, highlighting the data partitioning. The blue-shaded areas are used as a new sample. Further, **Tables 13** and **14** indicate that the dual momentum strategy has the best drawdowns.

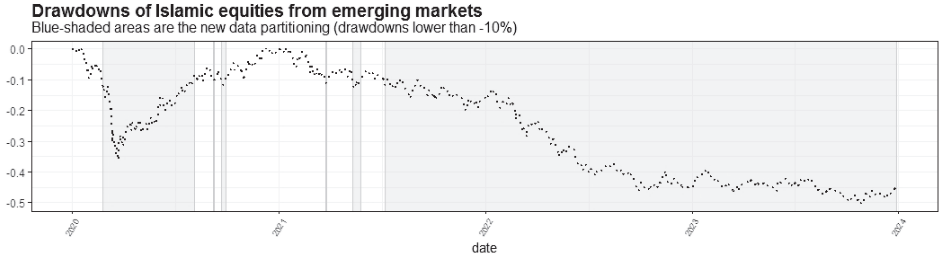


Figure 17.
New Data Partitioning (Emerging Markets)

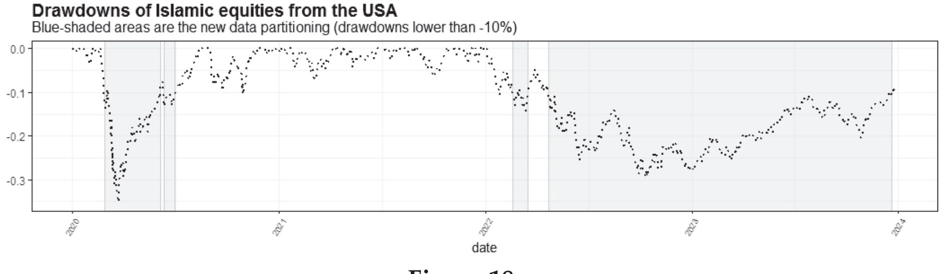


Figure 18.
New Data Partitioning (the USA)

Table 13.
Portfolio Performance (the USA) – New Data Partitioning

	Naive	Dual Momentum	Min. Variance	Risk Parity	All Equities	Passive
Mean returns %	8.754	7.283	0.195	3.628	22.272	6.300
Std. deviation %	7.266	8.181	6.204	6.205	13.645	6.478
Downside Volatility %	0.309	0.351	0.252	0.251	0.603	0.265
Minimum returns %	-1.085	-1.523	-1.564	-0.921	-1.887	-1.122
Maximum returns %	1.507	1.872	1.961	1.690	2.225	1.865
Skewness	0.327	0.272	0.721	0.719	0.039	0.720
Kurtosis	0.234	1.406	3.836	1.258	-0.324	2.177
Sharpe %	5.177	3.793	0.147	2.697	6.703	4.633
Sortino %	12.028	8.590	0.308	5.938	15.926	9.993
Omega	1.215	1.180	1.006	1.100	1.299	1.177
Value-at-Risk %	-0.671	-0.762	-0.527	-0.534	-1.318	-0.540
Median drawdown %	1.932	1.273	1.305	1.430	3.349	1.428
Maximum drawdown %	6.314	5.662	5.956	5.731	10.984	5.903

Notes: This table shows the portfolio performance. A positive skewness (favorable) typically has minor losses and few significant gains. The larger the risk-adjusted returns (Sharpe, Sortino, and Omega), the better the performance. The larger the drawdown, the more significant the decrease in an investment's peak-to-trough over time and the riskier the investment. Similarly, the larger the downside volatility, measured by semi-deviation, which assesses the below mean fluctuations, the riskier the investment. In addition, the lower the value-at-risk, a loss that we are pretty confident will not be surpassed if the portfolio is held for a certain period, the riskier the investment.

Table 14.
Portfolio Performance (Emerging Markets) – New Data Partitioning

	Naive	Dual Momentum	Min. Variance	Risk Parity	All Equities	Passive
Mean returns %	-11.251	-7.342	-9.528	-9.588	-20.795	-13.090
Std. deviation %	9.449	7.569	5.375	6.511	18.360	7.260
Downside Volatility %	0.413	0.326	0.219	0.272	0.811	0.310
Minimum returns %	-2.087	-1.911	-1.188	-1.482	-5.241	-1.702
Maximum returns %	2.114	2.421	1.943	1.703	3.712	1.829
Skewness	0.120	0.315	0.836	0.497	-0.017	0.335
Kurtosis	0.611	3.691	4.017	1.406	0.778	1.508
Sharpe %	-4.487	-3.972	-7.845	-5.940	-4.188	-6.972
Sortino %	-10.182	-8.577	-15.691	-12.916	-9.632	-15.281
Omega	0.826	0.820	0.733	0.786	0.831	0.743
Value-at-Risk %	-0.995	-0.734	-0.482	-0.641	-1.971	-0.745
Median drawdown %	13.504	2.974	20.855	11.062	21.991	14.074
Maximum drawdown %	26.206	17.011	20.855	21.531	43.106	27.584

Notes: This table shows the portfolio performance. A positive skewness (favorable) typically has minor losses and few significant gains. The larger the risk-adjusted returns (Sharpe, Sortino, and Omega), the better the performance. The larger the drawdown, the more substantial the decrease in an investment's peak-to-trough over time and the riskier the investment. Similarly, the larger the downside volatility, measured by semi-deviation, which assesses the below mean fluctuations, the riskier the investment. In addition, the lower the value-at-risk, a loss that we are pretty confident will not be surpassed if the portfolio is held for a certain period, the riskier the investment.

4.9. Analysis/Discussion

This section discusses the new approach and the alternative version of the dual momentum strategy. It then discusses the Sukuk performance for hedging compared to previous studies, which is still lacking in the current literature. In the last part of this section, we discuss the importance of this research to the literature on Islamic finance.

Table 1 presents the limitations of prior studies. This research relaxes the assumptions made in the previous studies. GARCH-based methods and quintile regression are widely used in the literature to evaluate the haven qualities of gold and Sukuk. However, those methods have limitations. For example, the quintile regression based on the dynamic correlation of DCC—GARCH does not show the haven qualities of haven assets at different quintiles. Thus, Kumar & Padakandla (2022) have developed the Wavelet Quintile Correlation (WQC) method to address the limitations of the methods used in the previous studies.

This study extends the Wavelet Quintile Correlation (WQC) method of Kumar & Padakandla (2022), incorporating nonstationary volatility. Specifically, this research uses the current state-of-the-art methodologies, including MSGARCH and TVGARCH. A dual-regime volatility is statistically more appropriate than a single-regime one. Furthermore, the TVGARCH model is also employed to enhance the quality of WQC. Additionally, this study enhances the original WQC by using the inflation-adjusted returns, avoiding the illusion of money.

This study shows that Sukuk can be a haven asset. This finding contrasts with the previous studies that show Sukuk is not a haven asset (Naeem et al., 2023; Nugroho & Kusumawardhani, 2023; Qadri et al., 2024). The possible reason is that previous studies do not consider volatility dynamics.

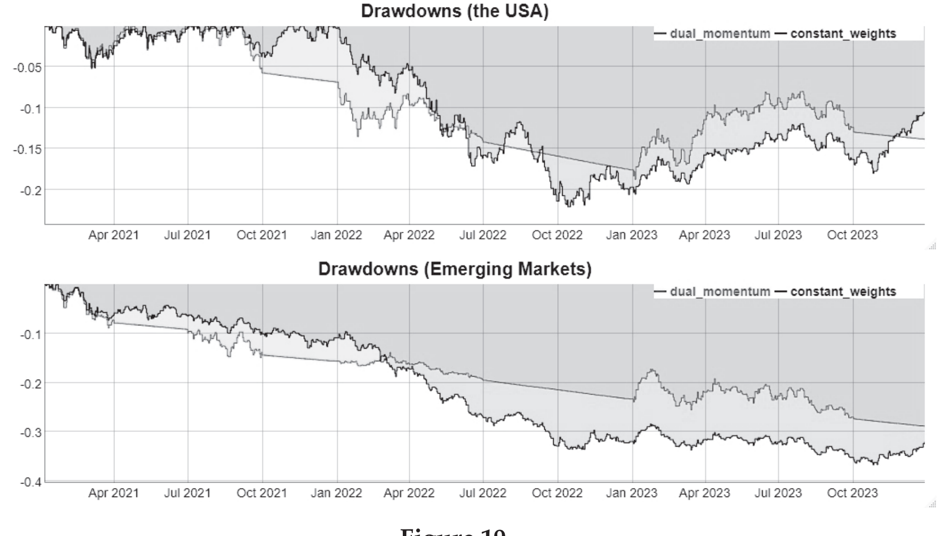


Figure 19.
Drawdowns of Dual Momentum Versus Constant Weight

Further, this study develops another version of the dual momentum strategy based on the results from MSGARCH-WQC. We rebalance the portfolio quarterly to adapt to market dynamics while having a moderate portfolio turnover. This strategy provides a better drawdown than the constant weight strategy with yearly rebalancing (45% Gold, 45% Equities, 10% Gold) proposed by Vliet & Lohre (2023). The transaction cost is 63 basis points.

Moreover, this research extends Masih et al. (2018) review of the current quantitative studies on Islamic equities, such as the performance of Islamic equities relative to non-Islamic ones and Socially Responsible Investing or SRI (Abdelsalam et al., 2014; Charfeddine et al., 2016; Charles et al., 2015; Jawadi et al., 2014), the performance of Islamic equities during a bear market (Ajmi et al., 2014), the diversification benefits of Islamic equities in international portfolio (Majdoub & Mansour, 2014), the analysis of Islamic equities risk (Bekri & Kim, 2015), calendar anomalies (Abbes & Abdelhédi-Zouch, 2015), and the overview of Shariah-compliant equities screening parameters (Clarke, 2015).

V. CONCLUSION AND RECOMMENDATION

5.1. Conclusion

This research modifies the Wavelet Quintile Correlation (WQC) method and then develops a trading strategy involving Islamic assets (Gold, Sukuk, and Islamic equities). Mainly, this study combines nonstationary volatility models (Markov-switching GARCH and Time-varying GARCH) with the conventional WQC to allow for the volatility dynamics. The daily data are four exchange-traded funds: Dow Jones Global Sukuk, Wahed FTSE USA Shariah, MSCI Emerging Market Islamic, and SPDR Gold. The return series are adjusted to inflation to avoid the money illusion. The results show that Sukuk and Gold are haven assets.

Next, this study optimizes the Sukuk—Gold—Islamic equities portfolio. We implement several widely used methods, such as naïve (the benchmark), minimum variance, risk parity, passive investing (70% Sukuk—30% Islamic equities), and the trend-following strategy (dual momentum). The rebalancing frequency is quarterly to minimize transaction costs. The results show that the dual momentum strategy has the best drawdown for extreme declines in the equities markets.

5.2. Recommendation

The results of this research can be beneficial for both academics and investors. This research enhances the WQC model recently proposed by Kumar & Padakandla (2022). For investors, the recommended trading strategy for the Sukuk – Gold – Islamic equities portfolio is the dual momentum model with quarterly rebalancing. However, transaction costs may greatly impact portfolio performance. The proposed strategy is only preferable when the transaction cost is below 70 basis points (the average transaction cost for ETF is 63 basis points). The naive strategy is suggested when the transaction cost exceeds 70 basis points. In other words, this research suggests assigning a relative value to each asset class (Sukuk, Gold, and Islamic equities) based on how well it has performed over the past three months relative to other assets in the same class (Sukuk and Gold are havens, in the strategy they are in the same class) and whether or not it has had a positive return. As long as the top-performing asset in the asset class has a positive return above zero, dual momentum invests in those assets. Otherwise, the allocation is shifted to cash.

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