Journal of Islamic Monetary Economics and Finance, Vol. 12, No. 1 (2026), pp. 59 - 80
p-ISSN: 2460-6146, e-ISSN: 2460-6618

OPTIMIZING ISLAMIC PORTFOLIO FORMATION USING MATHEMATICAL AND SHARIAH APPROACHES

Abdul Aziz[1*] and Abdurakhman Abdurakhman[2]

[1] Department of Statistics, Jenderal Soedirman University, Indonesia; [2] Department of Mathematics, Universitas Gadjah Mada, Indonesia

ABSTRACT

This paper introduces the Best Sharia-based Capital Asset Pricing Model (BSCAPM), a mathematical modification of the BCAPM model integrating Islamic finance principles. The study focuses on optimizing the beta in the model, incorporating factors aligned with Islamic principles, such as zakat and purification, while excluding short selling. Using data from the Jakarta Islamic Index (JII) from June 2020 to May 2024, the BSCAPM portfolio outperforms the BCAPM portfolio in terms of the Sharpe ratio. The results suggest that BSCAPM could serve as an effective alternative for modeling in Islamic investments, providing Muslim investors with a Shariah-compliant, optimal portfolio formation model. The research contributes to the underexplored domain of portfolio selection modeling in the Islamic sector, enriching references on asset pricing of Shariah portfolios, particularly in the Indonesian Shariah stock market.

Keywords: Portfolio, Best beta, SCAPM, Zakat, Purification.
JEL classification: C02; C13; C52; **G11; G12**.

Article history:
Received : August 5, 2024
Revised : November 5, 2024
Accepted : February 26, 2026
Available online : March 31, 2026

https://doi.org/10.21098/jimf.v12i1.2300

* **CONTACT** Abdul Aziz: abdul.math@unsoed.ac.id; Jenderal Soedirman University, Purwokerto, Banyumas, Jawa Tengah 53122, Indonesia

I. INTRODUCTION

In the current era of globalization, capital markets worldwide have experienced very rapid development. Indonesia is no exception. Based on data obtained from the IDX, the number of publicly listed stocks in Indonesia as of May 10, 2024, was 852, spanning various sectors. For investment, investors often consider two primary factors: potential profits (returns) and the risks associated with uncertainty. The risk measure used in CAPM is beta, representing systematic risk. Beta is widely utilized for two reasons. First, it enhances total risk measures by avoiding reliance on variance and standard deviation. Second, various empirical studies suggest that beta is relatively stable. In this study, we introduce a slight variation to the beta in the Capital Asset Pricing Model (CAPM) by incorporating risk as the second moment of the portfolio return around the target return. The objective of this analysis is to enhance accuracy by minimizing pricing errors in the CAPM. By fitting US historical data into the model, it is found that optimizing beta could improve the pricing accuracy of the CAPM by 20% to 30% per year (Zou, 2006). Although CAPM can enhance pricing accuracy, it has inherent shortcomings in practice due to its assumptions and limitations as a single-factor model. Consequently, numerous authors have developed and adapted CAPM to real-world conditions. The presence of interest rates is evident in lending and borrowing activities, along with their associated risks. Indeed, during periods of instability, risks may either rise or fall. This phenomenon leads to the concept of Sharia as a solution to overcome problems in the financial system.

In alignment with the growing trend of Islamic investment, many individuals are transitioning to the Islamic system. Several studies posit that Islamic finance is more reliable than conventional finance. This perspective is based on evidence indicating that Islamic banking yields positive results in three critical aspects: profitability, efficiency, and liquidity. Indeed, Parashar & Venkatesh (2010) reports that even in crisis conditions, Islamic banking demonstrates greater resilience compared to conventional banking. Additionally, Islamic Banking (IB) has proven to be more profitable than conventional banking (CB) based on research results spanning from 2007 to 2010.

Some researchers agree that the Capital Asset Pricing Model (CAPM) can be applied in Islamic finance (Hakim et al., 2016; Hammami & Oueslati, 2017; Selim, 2008). One of the elements in CAPM requires a risk-free interest rate, commonly known as the risk-free return. Nevertheless, the concept of a risk-free interest rate is incongruent with Sharia, as Islam prohibits the payment or receipt of interest. El-Ashker (1987) modifies the traditional Capital Asset Pricing Model (CAPM) by replacing the risk-free rate of return with a zakat rate of 2.5% in asset pricing. This adjustment is cited as a valuable reason for Islamic investors to consider investment. Recently, Derbali et al. (2017) and Hazny et al. (2020) attempt to include sukuk, zakat, and purification in the process. Sukuk from the Islamic Development Bank (IDB) can be a novel breakthrough in Islamic investment, despite the absence of a risk-free interest rate. The next question is: what is the difference between sukuk rates and interest rates? Sukuk structures are designed to circumvent the religious prohibition of usury. In a traditional bond structure, the issuer pays interest to the bondholder (Derbali et al., 2017). Sukuk can be an investment in various types of assets, particularly in ventures devoid of prohibited *riba* (a risk-free interest rate)

components and non-halal businesses. This modification of CAPM is referred to as the Sharia Compliant Asset Pricing Model (SCAPM) in this study. The three principles included in this model are the replacement of interest rates with sukuk, the incorporation of zakat deduction, and the consideration of purification.

In this study, CAPM is modified using two methods simultaneously to obtain an enhanced portfolio formation model. The first modification aims to optimize beta, thereby minimizing pricing errors. The second involves replacing interest rates and includes Sharia elements related to zakat, sukuk, and purification.

Section 2 reviews the literature, covering key theories such as zakat and purification, the mean-variance model, the CAPM, and Shariah investment in Indonesia. Section 3 describes the data, the model development process, and the research methods employed. In Section 4, a thorough analysis of the results is presented, and Section 5 wraps up with a summary of the findings and recommendations.

II. LITERATURE REVIEW

2.1. Background Theory

2.1.1. Beta of the Stock

According to William Sharpe, the changes in a stock's profit are influenced partly by the stock's specific characteristics and partly by general economic conditions. Sharpe suggests that these changes can be estimated by analyzing the changes in the profits of all stocks in the capital market. Beta is a measure of the volatility of returns on a security or portfolio relative to the market returns. The beta of security i measures the volatility of that security's returns in relation to market returns. Therefore, beta is a measure of risk relative to market risk.

Volatility refers to the fluctuation of returns on a security or portfolio over a specified period. If the returns of a security or portfolio systematically mirror the fluctuations of market returns, the beta of that security or portfolio is 1 (Bodie et al., 2021). In other words:

a. A beta value of 1 indicates that the security's returns fluctuate in direct alignment with market returns.
b. A beta value greater than 1 is associated with a "high-risk stock" or "aggressive stock," meaning the stock experiences relatively high fluctuations. Investors typically purchase this stock with the goal of achieving short-term profits.
c. A beta value less than 1 is associated with a "low-risk stock" or "defensive stock." This stock tends to be more stable during recessions or uncertain economic conditions and offer consistent dividends and income regardless of market performance.

To measure the systematic risk of each stock, the beta coefficient (β) is used. The formula for beta is as follows:

$$\beta = \frac{E\left[(r_m - E(r_m))x\right]}{E\left[(r_m - E(r_m))x_m\right]} = \frac{cov(r_m, r)}{Var(r_m)} = \frac{\sigma_{r_m, r}}{\sigma^2_{r_m}}, \quad (1)$$

where :
$Cov(R_m, R)$ = Covariance between the security's return and the market's return.
$Var(R_m)$ = Variance of the market's return.

2.1.2. Best Beta Capital Asset Pricing Model

2.1.2.1. Best Beta Estimation

The assumption of multivariate normality ensures that the risk-efficient optimal portfolio has the highest Sharpe ratio among all assets and portfolios. Additionally, the multivariate normal distribution indicates that the two-fund split holds for all investors only if each asset's return satisfies the two-fund separation condition.

In order to optimize the Best Beta Capital Asset Pricing Model by minimizing pricing errors, we can employ the following density function:

$$f\left(x_t \mid x_{m,t}\right) = \left(2\pi\right)^{N/2} \left|\Sigma\right|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\left(x_t - \beta x_{m,t}\right)^T \Sigma^{-1}\left(x_t - \beta x_{m,t}\right)\right\} \tag{2}$$

Then, the above equation is estimated using the Maximum Likelihood Estimation (MLE) method with the likelihood function as follow:

$$\begin{aligned} L(\beta,\Sigma) &= \prod_{t=1}^{T} f(x_t \mid x_{m,t}) \\ &= \prod_{t=1}^{T} (2\pi)^{-\frac{N}{2}} |\Sigma|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(x_t - \beta x_{m,t})^T \Sigma^{-1}(x_t - \beta x_{m,t})\right\} \\ &= (2\pi)^{-\frac{NT}{2}} |\Sigma|^{-\frac{T}{2}} \exp\left\{\sum_{t=1}^{T} -\frac{1}{2}(x_t - \beta x_{m,t})^T \Sigma^{-1}(x_t - \beta x_{m,t})\right\}. \end{aligned}$$

Taking the natural logarithm and then estimating the function, partial derivatives of β and Σ are obtained:

$$\hat{\beta} = \frac{\sum_{t=1}^{T} x_t x_{m,t}}{\sum_{t=1}^{T} x_{m,t}^2} \tag{3}$$

and

$$\Sigma^{-1} = \frac{1}{\left[\sum_{t=1}^{T} (x_t - \beta x_{m,t})(x_t - \beta x_{m,t})^T\right]}. \tag{4}$$

The role of the best beta denoted as β^B is crucial in minimizing mispricing. Utilizing the estimation process, we obtain the optimal beta for the CAPM model:

$$\beta^B = \frac{E(x_m, x)}{E(x_m^2)} = \frac{cov(x_i, x_m) + E(x_i, x_m)}{var(x_i) + [E(x_i)]^2} \tag{5}$$

The Best Beta CAPM model aims to predict and explain asset returns more accurately than the traditional CAPM model.

2.1.2.2. BCAPM Weigh Formation

Investors combine a risky asset with a risk-free asset. An asset is considered risk-free when the future return is certain. One example of a risk-free asset is government-issued bonds. In the case of Indonesia, Bank Indonesia Certificates (SBIs) issued by Bank Indonesia serve as an example of a risk-free asset.

The portfolio weighting formula can be solved by minimizing the variance ($w^T \Sigma w$) with the constraint $w^T (\mu - R_f 1) = \mu_p - R_f$, which can be expressed as:

$$\begin{aligned} &\min \quad w^T \Sigma w \\ &\text{s.t} \quad w^T \mu = \mu_p \\ &\text{and} \quad w^T 1 = 1 \end{aligned}$$

By using the Lagrange function, the weighting equation for BCAPM is obtained.

$$w_i = \frac{\Sigma^{-1}(\mu - R_f \mathbf{1})}{(\mu - R_f)\Sigma^{-1}\mathbf{1}^T} \tag{6}$$

where R_f is risk free rate, Σ is varian covarians matrix, and **1** vector with all its elements set to one.

2.1.3. Shariah Compliant with Zakat, Sukuk and Purification

Zakat, as one of the five pillars of Islam, is highly significant for a Muslim. The command to give zakat is mentioned repeatedly in the Quran. Studies on determining assets as zakat objects continue to evolve and are conducted by Islamic jurists (Krichene, 2012). Essentially, through zakat, it is hoped to assist the poor in reducing economic disparities. Zakat embodies the fairness of Islamic principles towards others. Specifically, it involves collecting a portion of wealth from the affluent and redistributing it to the less fortunate, provided certain conditions are met. These conditions include reaching the nisab threshold, which is the minimum amount of wealth required for zakat to be obligatory, and possessing assets for a full year, known as haul.

Zakat consists of both zakat fitrah (obligatory charity given at the end of Ramadan) and zakat maal (obligatory almsgiving on wealth). The Holy Qur'an highlights the importance of zakat in various verses, such as At-Taubah: 103, Al-Baqarah: 267, An-Nur: 56, and Adz-Zaariyat: 19. These verses stress that zakat helps cleanse and purify the community, fostering inner peace and harmony. In the context of investments, profits from transactions are considered a type of wealth subject to zakat, specifically falling under the category of zakat maal.

Sukuk are Shariah-compliant securities in the form of certificates or ownership proofs, each having equal value and representing an integral or undivided share of the underlying asset. This type of security involves asset securitization that adheres to Shariah principles in the capital market. The underlying asset serves as the basis for sukuk issuance and can be tangible goods such as land, buildings, and development projects, or intangible assets like services or beneficial rights to assets. Sukuk are Islamic bonds that grant holders rights over the income stream derived from the assets backing the certificates. In other words, there are two types of bonds: conventional and Islamic. The use of ijarah contracts in sukuk is considered the most effective way to fulfill Shariah requirements. Firstly, it avoids the prohibition of interest, and secondly, it makes sukuk a tradable instrument (Mustofa, 2018).

Investors, particularly those participating in Indonesia's Shariah-compliant stock market, need to account for zakat calculations on both short-term and long-term investments (Rahman, 2015). Shariah principles are essential for Muslim investors, shaping the design of investment products such as mutual funds that comply with Shariah guidelines. The Islamic stock market has gained prominence since the introduction of Islamic indices such as the Dow Jones Islamic Stock Index (DJII), which have demonstrated slightly above-average performance trends. Asset selection in this market follows various Shariah rules (Hassan et al., 2019).

Zakat plays a vital role in the community and is required on assets that meet the nisab threshold, equivalent to 85 grams of gold, with a typical zakat rate of 2.5%. Zakat is also due annually (hawl) once the assets have been held for a year. Rahman (2015) and Alam et al. (2017) review zakat calculations for stock investments, recommending a 2.5% zakat rate based on the Hijri calendar. During the first International Muktamar on Zakat in Kuwait (29 Rajab 1404 or April 30, 1984), it was agreed that zakat on profits from stock investments could be calculated at rates of 5% or 10%, similar to zakat on agricultural yields (Rahman, 2015).

Purification is another crucial aspect of Shariah-compliant investment. It involves the process of cleansing business practices and ensuring that profits from trade or investment are derived from halal sources. This principle, emphasized by Albaity (2011), focuses on maintaining the integrity of earnings. As noted by Hazny et al., (2020), purification is different from zakat in that it aims to verify and purify earnings to prevent participation in non-halal activities.

In Indonesia, the purification process is crucial for identifying Shariah-compliant stocks. The MUI National Sharia Board has established guidelines for classifying stocks on the Indonesia Sharia Stock Index (ISSI), requiring that non-halal income does not exceed 10% of total income. Shariah investors should recognize that investing in Shariah-compliant stocks typically involves long-

term commitments, influenced by the size of the capital invested (Abdalloh, 2019). Larger investments generally lead to longer holding periods. Successful investments generate profits, and investors must also meet their zakat obligations.

2.1.4. Portfolio Modeling with Sharia Adjustment

The utilization of Shariah principles in economic and financial domains through theoretical and applied models is on the rise. Studies focusing on theoretical frameworks largely centers on adapting traditional models to align with Shariah-compliant assets. Research in Islamic investments, such as the works by Abdullahi (2018) and Hassan et al. (2019) has explored model advancements for asset valuation and portfolio construction.

The adjusted Capital Asset Pricing Models (CAPM), which include components related to the risk-free rate (SCAPM version 1 to SCAPM version 5), have been reviewed and applied using data from the Indonesian stock market (Subekti et al., 2020). Among these models, the one that uses the sukuk rate is found to be more effective than others when replacing the risk-free rate within the CAPM framework under Islamic finance framework. However, substituting the risk-free rate with sukuk requires reconsideration due to factors like zakat and purification in stock investments (Subekti et al., 2022). This adjustment encourages investors to meet their zakat obligations and thoroughly assess a company's income based on its underlying assets. This study adds to the field by introducing various models for asset valuation and portfolio construction based on Shariah principles. The developed CAPM model is shown in the following equation:

$$R_i = R_f + \beta_i (R_M - R_f). \tag{7}$$

Furthermore, the equation is developed in accordance with Sharia principles:

1. SCAPM version 1: Removing Risk Free Rate (R_f) from CAPM (Tomkins et al., 1987).

$$\mu_i = \beta_i \mu_M . \tag{8}$$

2. SCAPM version 2: R_f replaced with the value of the zakat amount (Ashker, 1987).

$$\mu_i = R_{zakat} + \beta_i (\mu_M - R_{zakat}). \tag{9}$$

3. SCAPM version 3: R_f is replaced by the Nominal Gross Domestic Product (NGDP) index (Shaikh, 2009).

$$\mu_i = R_{NGDP} + \beta_i (\mu_M - R_{NGDP}). \tag{10}$$

4. SCAPM version 4: R_f replaced with inflation index (Hanif, 2010).

$$\mu_i = R_{Inflation} + \beta_i(\mu_M - R_{Inflation}). \tag{11}$$

5. SCAPM version 5: R_f replaced with sukuk index (Hakim et al., 2016).

$$\mu_i = R_{Inflation} + \beta_i(\mu_M - R_{Inflation}). \tag{12}$$

The development of SCAPM version 6 is henceforth referred to as the Shariah Compliant Asset Pricing Model (SCAPM) in this study. Three principles covered in this model include replacing R_f with sukuk, involving zakat reduction, and considering purification. The equation for SCAPM is as follows:

$$\mu_i = \frac{1}{1-\delta_i} R_{sukuk} + \hat{\beta}_i \left(\mu_M - \frac{R_{sukuk}}{1-\delta_M} \right), \tag{13}$$

where δ_M = purification factor.

III. DATA AND METHODOLOGY

3.1. Data

This research utilizes data pertaining to the shares of 70 publicly listed companies within the Jakarta Islamic Index (JII). The selection of JII 70 stocks is based on availability of data for the desired year published. The study sources the data on the JII 70 stocks from the Indonesia Stock Exchange (IDX) Statistical Report. The Covid-19 pandemic has affected markets globally, including the stock market in Indonesia. However, unlike the conventional stock market, which experienced a correction to its lowest point in the last decade, the Sharia-compliant stock market actually witnessed an increase during the pandemic. This aligns with research by Parashar & Venkatesh, (2010), which finds that Islamic finance is more stable during times of crisis. IDX also report that during the Covid-19 pandemic, Shariah investors in Indonesia increased by 25.2%, with a total of 424 Shariah-compliant stocks listed on the IDX, accounting for 59.5% and a market capitalization reaching 3,344.9 trillion Rupiah. Therefore, this study uses data from Sharia-compliant stocks listed on the IDX from 2020 to 2023. The investigation tracks the progression of JII 70 stocks over a span of 3 years, divided into seven periods. The data utilized consists of the weekly closing prices of 70 stocks from the JII index, spanning from June 1, 2020, to November 30, 2023. The selection of stocks is based on the following. First, stocks that are consistent constituents in the JII 70 throughout the observation period are included, resulting in 41 stocks. Secondly, among these included stocks, those exhibiting a positive mean return value are chosen, leading to 28 stocks. Additionally, the chosen stocks are required to demonstrate normality and stationarity. The consideration of stationarity testing is crucial for

demonstrating that the regression analysis is conducted on a stationary series. The outcomes of the normality and stationarity tests for stocks exhibiting positive returns are depicted in Table 1.

Table 1.
Jarque–Bera (JB) and Augmented Dickey–Fuller (ADF) Test

Stock	Company	Mean	JB test (p-value)	ADF test (p-value)
ADRO	Astra International Tbk.	0.00655	0.0015	0.01
AKRA	AKR Corporindo Tbk	0.00697	0.0000	0.01
ANTM	Aneka Tambang Tbk	0.00652	0.0000	0.01
ASII	Astra International Tbk	0.00198	0.3943	0.01
BMTR	Global Mediacom Tbk	0.00176	0.0000	0.01
BRIS	Bank Syariah Indonesia Tbk	0.00909	0.0000	0.01
CTRA	Ciputra Development Tbk	0.00391	0.0000	0.01
DMAS	Puradelta Lestari Tbk	0.00274	0.0000	0.01
ERAA	Erajaya Swasembada Tbk	0.00258	0.0000	0.01
ICBP	CBP Sukses Makmur Tbk	0.00167	0.002	0.01
INCO	Vale Indonesia Tbk	0.00271	0.0000	0.01
INDF	Indofood Sukses Makmur Tbk	0.00125	0.0209	0.01
ISAT	Indosat Tbk	0.00935	0.0000	0.01
ITMG	Indo Tambangraya Megah Tbk	0.00947	0.0000	0.01
JPFA	Japfa Comfeed Indonesia Tbk	0.00166	0.0000	0.01
KLBF	Kalbe Farma Tbk	0.00115	0.0011	0.01
LPPF	Matahari Department Store Tbk	0.00165	0.0003	0.01
LSIP	London Sumatra Indonesia Tbk	0.00196	0.0000	0.01
MAPI	Mitra Adiperkasa Tbk	0.0048	0.4295	0.01
MIKA	Mitra Keluarga Karyasehat Tbk	0.00079	0.2027	0.01
MYOR	Mayora Indah Tbk	0.00074	0.0000	0.01
PGAS	Perusahaan Gas Negara Tbk	0.00234	0.0000	0.01
PTBA	Bukit Asam Tbk	0.00460	0.0000	0.01
PWON	Pakuwon Jati Tbk	0.00064	0.0009	0.01
SMRA	Summarecon Agung Tbk	0.00177	0.0000	0.01
TLKM	Telkom Indonesia Tbk	0.00171	0.1721	0.01
TPIA	Chandra Asri Pacific Tbk	0.00257	0.0000	0.01
UNTR	United Tractors Tbk	0.00389	0.0000	0.01

Note that the results of the Augmented Dickey-Fuller (ADF) test in column 6 reject the null hypothesis of a unit root at a 5% significance level, confirming that all return series are stationary, while only few stocks having their returns to be normally distributed. From the Table, only five stocks, namely ASII, INDF, MAPI, MIKA, and TLKM, satisfy the criteria for both normal distribution and stationarity. All these stocks exhibit a standard deviation within the range of 2.7% to 5.46%. Among them, MAPI displays the highest risk, while INDF has the lowest risk. Additionally, MAPI boasts the highest mean return at 0.48%, while MIKA has the lowest mean return at 0.078%. Notably, INDF demonstrated superior

performance with a mean return of 0.124%, indicating the best balance of return and risk compared to the other stocks. Detailed summary statistics for the selected stocks are presented in Table 2.

Table 2.
Descriptive Statistics

Stock	Company	Sector	Mean	Std. Dev
ASII	Astra International Tbk.	Misc. Industries	0.001979	0.03567
INDF	Indofood Sukses Makmur Tbk	Consumer Non-Cyclicals	0.001245	0.02724
MAPI	Mitra Adiperkasa Tbk.	Non-Primary Consumer Goods	0.004805	0.05460
MIKA	Mitra Keluarga Karyasehat Tbk	Healthcare	0.000789	0.04459
TLKM	Telkom Indonesia Tbk.	Infrastructure	0.001709	0.03307

The other data required in this study is the market return represented by the Jakarta Composite Index (JCI). JCI data are from Yahoo Finance. The expected market return of 0.194% with a standard deviation of 1.781% is obtained based on the period June 2020 - May 2024. While the average weekly sukuk yield is 0.1101%, and the risk-free return is 0.0839%.

3.2. Model Development

Numerous studies on Islamic investment principles have emerged, where some place a particular emphasis on Shariah-compliant financial products. For example, sukuk is introduced as an alternative asset to conventional bonds in the Shariah capital market, differing fundamentally from traditional bonds (El Tiby & Grais, 2014). The adjustment in the Capital Asset Pricing Model (CAPM) involves incorporating the return rate of sukuk rather than the interest rate, aligning with Islamic principles that emphasize the association of profitability with risk (Hakim et al., 2016). The development of the CAPM within Shariah principles integrates elements such as zakat reduction and purification into the calculation of investment returns (Derbali et al., 2017; Hazny et al., 2020). The proposed model suggests using the yield rate of sukuk assets in place of the traditional risk-free rate. Consequently, this Shariah-compliant model necessitates incorporating zakat obligations and purification from the outset in calculating expected returns.

Two forms of investment reduction involve zakat and purification. Zakat, obligatory for Muslims when specific criteria are met, functions as a tax that serves to purify the wealth of Muslims. However, when engaging in investment activities, it is essential to consider the presence of prohibited activities. Therefore, purification becomes a crucial step in ensuring the ethical reception of profits. The designated zakat amount in this study is set at 2.5%. Purification, on the other hand, involves calculating the ratio of total income derived from non-halal sources, where the proportion of non-halal income to total revenue must not exceed 10%. The MUI National Shariah Board establishes this guideline for classifying Shariah-compliant stocks. As per Hazny et al. (2012), this purification process is represented by δ_i.

$$\delta_i = \frac{\text{non-halal income}}{\text{total income}} \tag{14}$$

Subekti et al. (2020) revise the adjusted return considering zakat and purification expenses. To differentiate between conventional and Islamic portfolio notations, the anticipated return for the new Islamic portfolio is denoted as $E(R_{ps})$. This study operates under the limiting assumption that, in future periods, the projected portfolio return is redefined, incorporating deductions for zakat and being vigilant regarding non-halal income. The anticipated return and variance of each individual stock are Sharia-compliant (i), incorporating adjustments for zakat (z) and the purification (δ_i) factor.

Assuming that all investors take into account the zakat (z) and purification (δ_i) factors, expected return of the portfolio is calculated as follows:

$$\mu_{ps} = (1-z)\left(\sum_{i=1}^{n} w_i(1-\delta_i)\mu_i\right) \tag{15}$$

Equation (13) represents the portfolio's novel anticipated return, while the portfolio variance is expressed as:

$$\sigma_{ps}^2 = \sum_{i=1}^{n} w_i^2(1-z)^2(1-\delta_i)^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{j\neq i}^{n} w_i w_j(1-z)^2(1-\delta_i)(1-\delta_j)\sigma_{ij} \tag{16}$$

Furthermore, in consideration of the zakat and purification factors, the Sharpe ratio equation, as defined by the measure of portfolio performance, is expressed as θ_{SR}:

$$\theta_{SR} = \frac{\mu_{ps} - (1-z)R_s}{\sigma_{ps}}$$

By substituting equations (15) and (16), we obtain the following equation:

$$\theta_{SR} = \frac{\sum_{i=1}^{n} w_i(1-z)(1-\delta_i)\mu_i - (1-z)R_s}{\sqrt{(1-z)^2\left(\sum_{i=1}^{n} w_i^2(1-\delta_i)^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{j\neq i}^{n} w_i w_j(1-\delta_i)(1-\delta_j)\sigma_{ij}\right)}}$$

Hence, the updated goal is to derive the SCAPM equation by maximizing the portfolio's new Sharpe ratio. This involves finding the derivative of the aforementioned equation above and acquiring the result:

$$\mu_i = \left[\frac{1}{1-\delta_i}\right] R_s + \beta_i \left[\mu_M - \frac{R_s}{1-\delta_M}\right] \tag{17}$$

The purification factor for the market, denoted as δ_M, plays a crucial role in enhancing market purification. Achieving market purification involves introducing a novel definition for the Sharpe Ratio concerning market returns. As a result, the objective function for the market portfolio is expressed through the subsequent equation:

$$\theta^* = \frac{(1-z)(1-\delta_M)\mu_M - (1-z)R_s}{\sigma_M} \tag{18}$$

This results in the formulation of the equation for market purification rate as:

$$\delta_M = 1 - \frac{\theta^* \sigma_M + (1-z)R_s}{(1-z)\mu_M} \tag{19}$$

In this research, our focus lies on integrating zakat and purification into the BCAPM for Shariah-compliant stock portfolios. The BCAPM is adapted to accommodate Shariah-compliant stock portfolios, and its modified procedure is emphasized. The SCAPM return serves as a benchmark for market asset returns. Building on the insights of Lai & Xing (2008) and Ruppert & Matteson (2015), the CAPM can be related to a regression model. Therefore, we approach the SCAPM through regression analysis, and it can be expressed in regression form as follows:

$$R_{it} - \left[\frac{1}{1-\delta_i}\right] R_{st} = \alpha_i + \beta_i \left(R_{Mt} - \frac{R_{st}}{1-\delta_M}\right) + \varepsilon_{it} \tag{20}$$

We have a market series (R_{Mt}), along with the return of each asset (R_{it}) in the actual dataset. The sukuk rate, R_{st}, can be considered a constant profit rate. β_i denotes the beta of stock *i*, δ_M stands for the market purification rate, and εit is the error term associated with stock *i*. We define a novel equilibrium excess return for Shariah-compliant stock as π_s, assuming ε_π is normally distributed.

In the context of the SCAPM analogous to the CAPM, the hypothesis $H_0 : \alpha = 0$ asserts the validity of the SCAPM. Estimating returns for the BSCAPM model, incorporating adjustments for Shariah-compliant assets, follows a procedure akin to that of the original BCAPM. Moreover, given that SCAPM can be formulated as a regression equation, the optimization of the beta in SCAPM can be accomplished by utilizing the optimal beta, denoted as the best beta SCAPM through the equation as follow:

$$\mu_i = \left[\frac{1}{1-\delta_i}\right] R_s + \beta_i^B \left[\mu_M - \frac{R_s}{1-\delta_M}\right], \tag{21}$$

where:

$$\beta^B = \frac{E(x_m, x)}{E(x_m^2)} = \frac{cov(x_i, x_m) + E(x_i, x_m)}{var(x_i) + [E(x_i)]^2} \tag{22}$$

Moreover, in BSCAPM, the allocation is determined by minimizing risk, specifically assessed through variance. The constraints on the desired return are established based on investor preferences. The total allocation must sum up to 100%, and short selling is not allowed, requiring all weights to be greater than 0.

$$\begin{aligned} &\min \quad w^T \Sigma_s w \\ &\text{s.t} \quad w^T \hat{\mu}_s = \mu_{ps} n \\ &\qquad w^T 1 = 1 \\ &\text{and} \quad w \geq 0 \end{aligned}$$

By using the mathematical optimation, the weighting equation for Best Beta SCAPM is obtained.

$$w_i = \frac{\Sigma_s^{-1} \mu_s}{\mu_s \Sigma_s^{-1} 1_{ps}^T} \tag{23}$$

where :
μ_s : return from Shariah-compliant stocks adjusted for zakat and purification.
Σ_s : variance-covariance matrix for Shariah-compliant stocks that has been adjusted according to Shariah principles.

3.3. Method

To implement the updated SCAPM version 6 model, which integrates sukuk, zakat, and purification, adjustments are made to both the mean and variance to reflect zakat and purification requirements. The purification for each asset is calculated based on the ratio of non-halal income to total income. Following DSN MUI guidelines, this ratio must be less than 10%. Annual financial statements for each asset are available at https://www.idx.co.id.

Sukuk offers a secure investment option for investors, guaranteed to be halal by DSN-MUI. Consequently, the sukuk rate is used as a substitute for the risk-free rate. In Indonesia, sukuk types include Sukuk Ijarah, Sukuk Mudharabah, Sukuk Musyarakah, and Sukuk Istisna, with government sukuk categorized as Sukuk Tabungan (ST) and Sukuk Ritel (SR). Retail sukuk strictly adheres to Shariah principles, generating returns from investment profits. The sukuk rate

demonstrated significant year-on-year increases from 2020 to 2024. In terms of liquidity risk, sukuk typically maintains high liquidity shortly after issuance, though this can diminish if it ranks lower. Credit ratings of sukuk may be downgraded or face sporadic issuance from certain issuers (Krichene, 2012).

The optimal beta in the model is derived by refining the beta in BCAPM. This optimal beta is believed to minimize pricing errors (Zou, 2006), enabling the SCAPM model, which incorporates adjustments for Sharia principles, to create a more efficient portfolio. Additionally, to evaluate the performance differences between BCAPM and BSCAPM, the Sharpe ratio is used. This measure shows that BSCAPM performs better than BCAPM

IV. RESULTS AND ANALYSIS

4.1. BCAPM Calculation

We outline the computation of BCAPM, with the outcomes of beta BCAPM presented in Table 3. The significant test for α and β are given in Columns 4 and 5 of Table 3.

Table 3.
Alpha, Beta, and the Excess Return BCAPM

Stock	Alpha	Beta	Pval(Alpha)	Pval(Beta)	BCAPM
ASII	-0.00923	0.99672	0.9786	8.52×10^{-13}	0.002043
INDF	-0.00923	0.44045	0.8588	8.79×10^{-05}	0.00136
MAPI	-0.00923	0.8496	0.4307	2.19×10^{-04}	0.001862
MIKA	-0.00923	-0.1506	0.7387	3.98×10^{-02}	0.000647
TLKM	-0.00923	0.9751	0.8902	2.39×10^{-14}	0.002016

The validity of the BCAPM model for the five chosen stocks is confirmed as each p−value $\geq$ 0.05 satisfies the criteria. Additionally, the significance of the relationship between the return of each asset and the market return is established, as indicated by the p-value $\leq$ 0.05. The information presented in the table indicates that each asset's optimal beta is less than 1, signifying a low-risk stock. This implies that during a recession or economic downturn, each stock is generally stable concerning dividends, earnings, and overall market performance. Based on the expected returns, ASII is the top-performing stock with a return rate of 0.2043%, surpassing all others. In contrast, MIKA stock offers the lowest return at 0.0647%.

4.2. BSCAPM Calculation

The inclusion of zakat and purification variables in the SCAPM model leads to a reduced anticipated portfolio return compared to the CAPM model (Subekti, Abdurakhman, & Rosadi, 2022). This is a direct consequence of the underlying assumption that the expected return diminishes and the variance contracts. Detailed information for purifying each asset is available in the appendices of respective company financial statements. As per the 2024 financial statements, the purification rate is determined based on the provided data.

In the framework of SCAPM, we determine the purification differential for each asset by consulting the annual financial report. In Indonesia, a stock can be classified as Shariah-compliant only if its purification level is below 10%. Accounting professionals are essential in calculating the exact purification level of assets based on annual financial statements. The expected return of SCAPM is illustrated in Table 4.

Table 4.
Beta Estimation, Purification and Return SCAPM

Stock	δ_i	$\frac{R_s}{1-\delta_i}$	Beta	SCAPM
ASII	0.00934	0.001361094	0.96085	0.002022
INDF	0.01201	0.001364772	0.42188	0.001656
MAPI	0.00225	0.001351422	0.79860	0.001902
MIKA	0.01509	0.001369040	-0.15358	0.001264
TLKM	0.00166	0.001356466	0.93588	0.002000

Upon initial examination, the anticipated return value of SCAPM appears nearly identical to the BCAPM value found in Table 3. Despite being distinct, the disparity between the expected return values of SCAPM and BCAPM is minimal. As indicated in the SCAPM study conducted by Hazny et al. (2020), this similarity can be attributed to the adjustment for zakat and purification. Similar to SCAPM, estimating the expected excess return of Best Beta SCAPM involves considering several values, including the amount of zakat, sukuk yield, stock purification rate, and market. Additionally, the beta calculation utilizes equation (17) to achieve more optimal results. The calculation results with varying beta values are presented in Table 5 below:

Table 5.
Estimation Best Beta, Purification and Return BSCAPM

Stock	δ_i	$\frac{R_s}{1-\delta_i}$	Best Beta	BSCAPM
ASII	0.00934	0.001361	0.99622	0.002045
INDF	0.01201	0.001365	0.44018	0.001667
MAPI	0.00225	0.001351	0.84857	0.001934
MIKA	0.01509	0.001369	-0.15082	0.001265
TLKM	0.00166	0.001356	0.97462	0.002025

4.3. Robustness Test

The modification of BCAPM by integrating Shariah elements like zakat, sukuk, purification, and the prohibition of short-sales and interest is part of the BSCAPM framework. As a result, additional robustness tests are performed on BSCAPM. Hazny et al., (2020) suggest that the Fama-MacBeth regression, specifically through second pass regression, can be employed to assess the validity of the BCAPM

model. The outcomes of the second-stage regression are displayed in Table 6, and the t-statistic is used to determine whether the coefficients significantly differ from zero.

Table 6.
Second Pass Regression

	Coefficient	Std. Error	t-statistic	p-value
λ_0	0.001635	0.000724	2.2584	0.10909
λ_1	0.0010209	0.000213	4.794	0.01727

The findings indicate that the intercept λ_0 has a value of 0.00163470, which is not significantly different from zero. This aligns with the BCAPM hypothesis, which suggests the intercept should not be statistically significant. On the other hand, the coefficient λ_1 is significantly different from zero, as the p-value (0.01727) is smaller than 0.05.

In summary, the results indicate that the BSCAPM model proposed in this paper is valid, as confirmed by tests conducted on data from the Indonesian Stock Exchange during the sample period.

4.4. Allocation of Assets within the Portfolio

In the preceding section, we obtain estimates for the anticipated returns of BCAPM, SCAPM, and BSCAPM. Now, we will determine the allocation of each stock. The process of optimizing weights through SCAPM is analogous to adjusting inputs in the mean-variance method, aiming for enhanced diversification outcomes. Furthermore, the beta employed in this approach represents the optimal beta. The outcomes obtained by applying the BCAPM, SCAPM, and BSCAPM approaches appear nearly identical upon initial observation, as illustrated in Table 7 and Figure 1.

Table 7.
Portfolio Weights for Each Method

Stock	BCAPM	SCAPM	BSCAPM
ASII	0.2601	0.2232	0.2237
INDF	0.2705	0.3176	0.3164
MAPI	0.0295	0.0160	0.0171
MIKA	0.1216	0.1691	0.1681
TLKM	0.3182	0.2742	0.2747

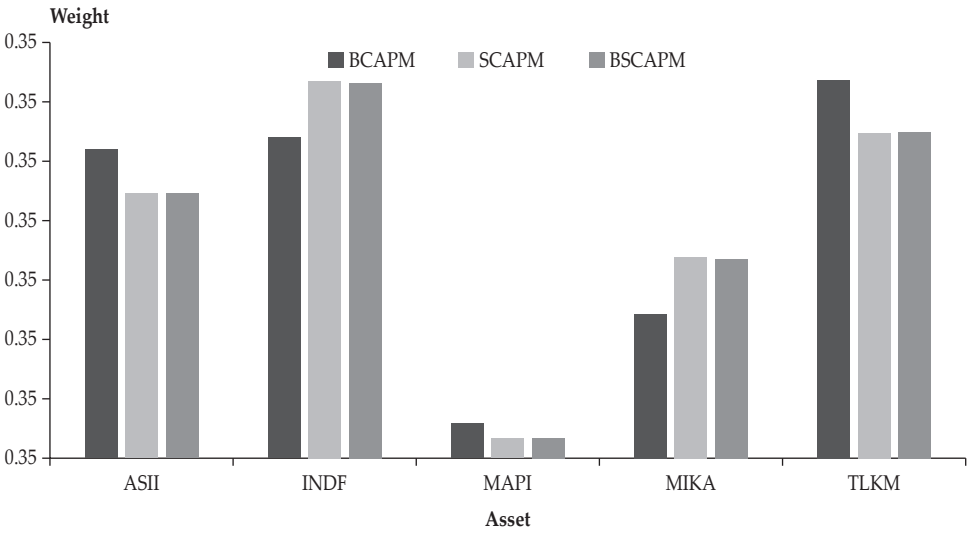

Figure 1.
Weight of All Portfolios

From the figure above, it can be observed that there are corrections in the weights of some stocks. MAPI still receives the smallest weight in the portfolio, both with the BCAPM and BSCAPM methods. The second correction occurs in TLKM shares, which, in the BCAPM model, is the stock with the largest proportion; however, in the BSCAPM model, INDF shares has the largest weight. ASII and MAPI experience the opposite correction, with a smaller weight for BSCAPM compared to the weight on BCAPM. Positive corrections are applied to INDF, MIKA, and TLKM stocks, resulting in larger weights than before. This weight change is an adjustment due to the input in the second stage of the BSCAPM method. The results of the weights on each asset do not contain negative weights, thus meeting the constraint without short selling. This is particularly important for Muslim investors adhering to Sharia principles.

4.5. Portfolio Performance

In this segment, we will present the outcomes of comparing two portfolios, wherein BCAPM is regarded as the traditional model, while BSCAPM is introduced as a novel alternative approach for allocating Islamic stocks. The weight outcomes derived from BSCAPM in Table 7 closely resemble those obtained through BCAPM. BSCAPM's advantage lies in its optimized beta, minimizing pricing errors and avoiding elements conflicting with Sharia principles during portfolio target return calculation. This approach sidesteps practices like interest usage, considers the impact of zakat on returns, and incorporates purification into a modeling framework aligned with Sharia principles. A comparison of the return, risk, and Sharpe ratio for the portfolios is presented in Table 8.

Table 8.
Portfolio Performance Results of BCAPM and BSCAPM

Portfolio	Expected Return	Risk	Sharpe Ratio
BCAPM	0.001677	0.019301	0.0438
BSCAPM	0.001787	0.01787	0.05353

It seems that the portfolio's return and risk are nearly identical when employing BSCAPM compared to BCAPM. The BSCAPM portfolio carries a slightly lower risk than BCAPM, potentially leading to an enhanced Sharpe performance. Specifically, the Sharpe ratio for BSCAPM stands at 5.35%, surpassing the 4.38% of BCAPM. In contrast to the BCAPM model, the portfolio formed using BSCAPM demonstrates a superior Sharpe ratio coupled with reduced risk.

A portfolio utilizing the BSCAPM demonstrates a higher Sharpe ratio compared to the BCAPM. When evaluating the Shariah-compliant version of the BCAPM against its conventional counterpart, it is observed that the BCAPM, incorporating zakat and purification, outperformed the standard BCAPM. The inclusion of purification and zakat in the model leads to a reduction in the modified expected return and variance for each asset. As a result, the portfolio's Sharpe ratio improves. This finding demonstrates that modifying the BCAPM benchmark leads to a better-performing portfolio in terms of asset diversification and overall performance. The shift from the traditional model to the Shariah version, characterized by zakat reduction and purification, results in changes to both the expected return and portfolio risks, thereby presenting a new Shariah-compliant strategy.

After determining the weight of each asset in the portfolio using the BCAPM and BSCAPM models, an investment simulation is conducted over the next 5 days with a capital of IDR 1,000,000,000.00. The data used in this simulation consist of daily closing stock prices as shown in the following table:

Table 9.
Stock Prices for the Next 5 Days

Date	ASII	INDF	MAPI	MIKA	TLKM
December 21, 2023	5550	6275	1715	2760	3950
December 22, 2023	5550	6275	1765	2800	3960
December 27, 2023	5550	6350	1750	2780	3940
December 28, 2023	5600	6450	1765	2770	3960
December 29, 2023	5650	6450	1790	2850	3990

After obtaining the stock price data for each stock, the fund proportions of the BCAPM and BSCAPM portfolios are established as shown in the following table:

Table 10.
Portfolio Fund Allocation

Stock	BCAPM		BSCAPM	
	Funds	Lots	Funds	Lots
ASII	IDR 260,100,000.00	462	IDR 221,400,000.00	393
INDF	IDR 270,500,000.00	431	IDR 314,500,000.00	501
MAPI	IDR 29,500,000.00	172	IDR 16,510,000.00	96
MIKA	IDR 121,600,000.00	443	IDR 164,200,000.00	599
TLKM	IDR 318,200,000.00	803	IDR 283,300,000.00	715

Table 11.
Portfolio Performance Simulation

Date	BCAPM		BSCAPM	
	Profit/Loss	%	Profit/Loss	%
December 21, 2023	- IDR 3,382,000.00	-0.34	- IDR 2,464,500.00	-0.25
December 22, 2023	IDR 3,435,000.00	0.34	IDR 3,591,000.00	0.36
December 27, 2023	IDR 482,500.00	0.05	IDR 985,500.00	0.1
December 28, 2023	IDR 8,041,000.00	0.8	IDR 7,950,000.00	0.8
December 29, 2023	IDR 8,693,000.00	0.87	IDR 9,142,000.00	0.91
Total	IDR 17,269,500.00	1.73	IDR 19,204,000.00	1.92

Based on Table 11 above, the investment results over 5 days show consistent gains, with the exception of the first day, where there is a loss of 0.34% for the BCAPM model and 0.25% for the BSCAPM model. The highest profit occurs on the fifth day for the BCAPM model, reaching 0.87%, while the BSCAPM model achieves its peak gain of 0.91% in the fifth day. The table also indicates that the total investment profit from the BSCAPM model exceeds that of the BCAPM model. Although the profit difference between the two models is modest, around 0.19%, the BSCAPM model aligns more closely with Sharia principles in creating an optimal stock portfolio.

V. CONCLUSION AND RECOMMENDATION

Recently, there has been increasing confidence in the global Islamic economic and financial system, paralleling advances in relevant research and Shariah finance in investments. However, the literature on Shariah modeling for portfolios is still insufficient. Integrating the Shariah system into the investment sector is expected to attract Muslim investors. This study aims to advance Shariah compliance in investments. Shariah principles, however, face some resistance, partly due to unfamiliarity with new terms such as riba (usury), which Islam strictly prohibits, including practices like risk-free rate. Other terms include zakat from income and purification in stock markets according to Shariah principles.

This research concentrates on two key aspects. Firstly, it seeks to enhance the BCAPM method by optimizing the beta in CAPM to minimize pricing errors. The research introduces a modification to the CAPM model by substituting the risk-

free interest rate with the zakat factor, incorporating purification, and excluding short sales. This leads to the development of a new optimal portfolio formation method called BSCAPM.

Applying the BSCAPM, zakat, and purification models to the Indonesian stock market shows that the BSCAPM model makes a corrective adjustment to the weightings used in the BCAPM model. Portfolios constructed using BSCAPM yield higher returns compared to those using BCAPM. Additionally, performance assessments using the Sharpe ratio demonstrate that BSCAPM outperforms BCAPM. These findings indicate that, despite adjustments in benchmarks due to zakat and purification, BSCAPM effectively delivers a superior portfolio in terms of asset diversification, expected return, and risk. This study offers valuable insights for investors, especially those following Islamic principles, to consider when investing in Islamic stocks.

Limitations of this study suggest potential avenues for future research. Evaluating the global applicability of the BSCAPM model would be beneficial, particularly its suitability in diverse international markets characterized by different economic conditions and regulatory environments. Furthermore, conducting sensitivity analyses could provide insights into how variations in crucial parameters, such as sukuk rates and purification criteria, influence the outcomes of the BSCAPM model. Developing a dynamic version of the model to accommodate fluctuations in zakat rates and purification criteria over time could enhance its realism and alignment with evolving Islamic finance principles. These enhancements would contribute to a more comprehensive understanding and application of the BSCAPM model across various contexts.

REFERENCES

Abdalloh, I. (2019). *Pasar modal syariah* (Third Edition). Elex Media Komputindo.

Abdullahi, S. I. (2018). Contribution of mathematical models to Islamic economic theory: A survey. *International Journal of Ethics and Systems*, *34*(2), 200-212.

Alam, M. M., Akbar, C. S., Shahriar, S. M., & Elahi, M. M. (2017). The Islamic Shariah principles for investment in stock market. *Qualitative Research in Financial Markets*, *9*(2), 132-146.

Albaity, M. S. (2011). Impact of the monetary policy instruments on Islamic stock market index return. *Economics Discussion Papers, No. 2011-26*, Kiel Institute for the World Economy (IfW), Kiel.

Ashker, A. A. F. E. (1987). *The Islamic business enterprise*. London: Croom Helm.

Bodie, Z., Kane, A., & Marcus, A. J. (2021). *Investments* (Twelfth Edition). United States of America: McGraw-Hill Education.

Derbali, A., El Khaldi, A., & Jouini, F. (2017). Shariah-compliant capital asset pricing model: New mathematical modeling. *Journal of Asset Management*, *18*(7), 527-537.

El Tiby, A. M. R. M., & Grais, W. M. (2014). *Islamic finance and economic development: Risk, regulation, and corporate governance*. New Jersey: John Wiley & Sons.

Hakim, S. A., Hamid, Z., & Mydin Meera, A. K. (2016). Capital asset pricing model and pricing of Islamic financial instruments. *Journal of King Abdulaziz University: Islamic Economics*, *29*(1), 21-39.

Hammami, Y., & Oueslati, A. (2017). Measuring skill in the Islamic mutual fund industry: Evidence from GCC countries. *Journal of International Financial Markets, Institutions and Money*, *49*, 15-31.

Hanif, M. (2010). Risk and return under Sharia framework: An attempt to develop Sharia compliant asset pricing Model-SCAPM. *Pakistan Journal of Commerce & Social Sciences*, *5*(2), 1-8. http://ssrn.com/abstract=1716821

Hassan, M. K., Aliyu, S., Paltrinieri, A., & Khan, A. (2019). A review of Islamic investment literature. *Economic Papers: A Journal of Applied Economics and Policy*, *38*(4), 345-380.

Hazny, M. H., Hashim, H. M., & Yusof, A. Y. (2012, September). Revisiting Markowitz's Mean Variance analysis: A review from shariah perspective. In *2012 International Conference on Statistics in Science, Business and Engineering (ICSSBE)* (pp. 1-6). IEEE.

Hazny, M. H., Mohamad Hasim, H., & Yusof, A. Y. (2020). Mathematical modelling of a shariah-compliant capital asset pricing model. *Journal of Islamic Accounting and Business Research*, *11*(1), 90-109.

Krichene, N. (2012). *Islamic capital markets: Theory and practice*. Solaris South Tower, Singapore: John Wiley & Sons.

Lai, T. L., & Xing, H. (2008). *Statistical models and methods for financial markets*. New York, NY: Springer New York.

Mustofa, L. (2018). The substance of the formal prohibition of the Riba: Islamic finance and the tie with the real economy. *IJISH (International Journal of Islamic Studies and Humanities)*, *1*(1), 56-68.

Parashar, S. P., & Venkatesh, J. (2010). How did Islamic banks do during global financial crisis?. *Banks & Bank Systems*, *5*(4), 54-62.

Rahman, S. (2015). Ethical investment in stock screening and zakat on stocks. *Journal of Islamic Finance*, *4*(1), 39-62.

Ruppert, D., & Matteson, D. S. (2015). *Statistics and data analysis for financial engineering* (2nd ed.). New York: Springer.

Selim, T. H. (2008). An Islamic capital asset pricing model. *Humanomics*, *24*(2), 122-129.

Shaikh, S. A. (2009). Corporate finance in an interest-free economy: An alternate approach to practiced Islamic corporate finance. *Journal of Islamic Banking and Finance, International Association of Islamic Banks*. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1527310

Subekti, R., Abdurakhman, A., & Rosadi, D. (2022). Can zakat and purification be employed in portfolio modelling?. *Journal of Islamic Monetary Economics and Finance*, *8*, 1-16.

Subekti, R., Abdurakhman, & Rosadi, D. (2020). Modified capital asset pricing model (CAPM) into sharia framework. *Journal of Physics: Conference Series*, *1581*(1), 012021. IOP Publishing.

Subekti, R., Abdurakhman, Rosadi, D., & Hasim, H. M. (2022). Integrating zakat and purification in a modified Black–Litterman model for Shariah-compliant stock portfolios. *Asian Economic and Financial Review*, *12*(7), 549-564.

Tomkins, C., & Karim, R. A. 'A. (1987). The Shari'ah and its implications for Islamic financial analysis: An opportunity to study interactions among society, organization, and accounting. *American Journal of Islam and Society*, *4*(1), 101-115.

Zou, L. (2006). The best-beta CAPM. *Applied Financial Economics Letters*, *2*(2), 131-137.