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OPTIMAL HEDGE RATIO OF SUKUK AND ISLAMIC EQUITY: A NOVEL APPROACH

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ABSTRACT

This research applies a novel model to compute a hedge ratio. Specifically, the model modifies volatility forecasts of an exponentially weighted moving average method to account for the *fat*-tailed distribution of returns. This simpler model aims to overcome the widely-known drawback of the complex GARCH models that a long daily return period is required to ensure the model's convergence. The data are Islamic exchange-traded funds: SP Funds Dow Jones Global Sukuk ETF, Wahed FTSE USA Shariah ETF, and iShares MSCI EM Islamic UCITS ETF. Sukuk act as a diversifier over the turmoil period since they are positively correlated with Islamic equity and their volatility is less than that of Islamic equity. This work also implements widely-used methods such as Dynamic Equicorrelation-GARCH, GO-GARCH, asymmetric DCC-GARCH, naïve approach, and linear regression. Two forms of data splitting and a rolling-window analysis are carried out to reduce data mining bias. All models generate one-step ahead forecasts of hedge ratios. Applying wavelet-transformed returns and utility analysis incorporating third and fourth moments, the proposed models produce better performance than the competing models. The results remain the same irrespective of different hedging instruments (precious metals) and asset classes.

Keywords: Exponentially weighted moving average, GARCH, Hedge ratio.
JEL classification: C46; C58; F31.

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I. INTRODUCTION

Religious and faith-based investments have been the topic of interest for researchers during the COVID-19 period (Arif et al., 2022; Dharani et al., 2022; Karim & Naeem, 2022; Naeem et al., 2023; Sisodia et al., 2022). Investors who choose faith-based investing frequently invest in equity, bonds, and other assets consistent with their moral or religious principles (Alam & Ansari, 2020). Sukuk holders receive a portion of the revenue generated from the underlying assets, and Sukuk indicates shares in ownership over assets (Ghaemi Asl & Rashidi, 2021). Because Sukuk act as a diversifier during turmoil periods, they offer faith-based or religious investors an opportunity to cope with the impact of economic shocks on their asset portfolios (Naeem et al., 2023; Nguyen et al., 2021).

Further, Islamic hedging is considered more favorable compared to traditional hedging for several reasons, including the frequency of profitable months, the minimum premium payment, and the maximum profit payment (Ismal, 2022). Theoretical distinctions exist between conventional and Islamic hedging, namely in the field of money market operations. Several vital differences can be identified (Injadat, 2018). First, in the context of Islamic finance, it is recommended that Islamic hedging practices adhere to the utilization of Islamic money market rates. In contrast, conventional hedging methods have the flexibility to employ any money market rate available. Second, in order to implement currency hedging, it is necessary to have an underlying transaction, which can be either a real or financial activity.

This research focuses on Sukuk and Islamic equity due to their unique characteristics. First, rating organizations assign Sukuk a risk rating in relation to the risk they pose. This is similar to how bonds are assessed. They can be traded on a few stock exchanges worldwide. Conversely, Sukuk denote a portion of ownership in relation to an asset, whereas bonds are thought of as debt instruments (Samitas et al., 2021). Second, it is asserted that Sharia-compliant stocks can draw investors seeking a lucrative return on their capital because they are less volatile than conventional stocks (Alam & Ansari, 2020). However, there is a scarcity of empirical research on the assessment of Islamic equity and Sukuk's hedging performance.

Further, the literature has previously devoted substantial attention to hedge ratios. Motivated by recent literature indicating that a simpler model has the same predictive accuracy as the complex GARCH models when estimating volatility (Stamos, 2022), this study proposes a novel approach to compute a hedge ratio. Inspired by Carol Alexander & Dakos (2023), it applies an exponentially weighted moving average (EWMA) with a volatility adjustment model.

This study's methodology is very different from other approaches. It removes certain inherent econometric limitations in the existing literature. Furthermore, while many earlier researchers use in-sample analysis, this study focuses on out-of-sample hedging performance. It also considers heterogeneous timing preferences and applies commonly-used frameworks to compute hedge ratios such as Dynamic Equicorrelation-GARCH, GO-GARCH, asymmetric DCC-GARCH, naïve approach, and linear regression with rolling-window frameworks. All models generate 1-step ahead forecasts. Therefore, this study's fundamental contribution is clear: using a unique approach to calculate the hedging ratio that works beyond some inherent econometric limitations in most of the current literature.

The paper is structured as follows. Following the literature in the next section, we explain the methodology, empirical results and analysis, and the conclusion.

II. LITERATURE REVIEW

This section consists of two parts. The first part explains the theoretical argument concerning a hedge, a diversifier and a safe haven asset. The second part outlines the methods that have been used in the literature to calculate the hedge ratio.

2.1. Definitions of Hedge, Diversifier, and Safe Haven

The following definition serves as a foundation for the ensuing analysis. If investors include in their portfolios an asset that lowers losses during periods of market turmoil or turbulence the impact of shocks would be lessened, thereby enhancing capital market stability. The primary purpose of bonds in a portfolio is to lower volatility. Even in cases where there is a positive correlation, bonds will still reduce portfolio volatility if their volatility is less than that of stocks (Ryan, 2021). We differentiate between a safe haven asset, a hedge and a diversifier asset before moving forward.

2.1.1. Hedge

"*A hedge is an asset that is uncorrelated or negatively correlated with another asset on average"* (Baur & Lucey, 2010). A hedge lacks the ability to minimize losses during periods of market turmoil or turbulence because the asset may show a positive correlation during these times and a negative correlation during normal periods.

2.1.2. Diversifier

"*A diversifier is an asset that is positively (but not perfectly correlated) with another asset on average*" (Baur & Lucey, 2010). As with the hedge, the diversifier lacks the ability to minimize losses in extremely unfavorable market conditions.

2.1.3. Safe Haven

"A safe haven is an asset that is uncorrelated or negatively correlated with another asset in times of market turmoil" (Baur & Lucey, 2010). Therefore, the correlation is either positive or negative during normal or favorable market conditions. Since the price of the safe haven asset increases when the price of any other asset or portfolio decreases, it compensates the investor for losses in extremely unfavorable market conditions.

In addition to declining yields over the past few decades, stock-bond investors have also profited from diversification across the two asset classes. The average correlation between stocks and bonds was positive throughout the 20th century, but prior to the 2000s, this was not the norm. Instead, we have witnessed a negative correlation between them. Increases in stock-bond correlation would impact the allocation of assets and expected returns as well as portfolio risk (Brixton et al., 2023). Further, the stock markets and Sukuk have a positive and significant correlation that suggests non-hedge characteristics (Naeem et al., 2023). Based on the explanation above, we expect Sukuk to be a diversifier asset.

2.2. Methods to Calculate Hedge Ratios

The modeling of hedge ratios is a significant research topic. This area of research has grown more complex, such as several sophisticated GARCH (generalized autoregressive conditional heteroscedasticity) models. Theoretically, complex models can achieve better predictive accuracy. However, recent studies show that the added advantage of the complex GARCH models over a simpler model has been found to be insignificant in daily data (Carol Alexander & Dakos, 2023). The main reason is that GARCH models need several years of daily data for estimation (Bauwens, Laurent, & Rombouts, 2006).

Table 1 summarizes recent literature on the hedge ratios. It is clear that this work offers a novel method to compute hedge ratios of faith-based funds. Also, most studies have evaluated the hedging performance using Ederington's approach, which has been heavily criticized. Therefore, this research evaluates hedging performance considering the third and fourth moments of hedged portfolio returns.

Further, this research uses the hedging theory (Kroner & Sultan, 1993). One of the simplest ways to reduce risk is that investors holding $1 in the equity market should sell or short $1 of a hedging instrument. Hence, the hedge ratio is 1 (naïve strategy). However, this approach is ineffective since the potential risk for each investment varies as the market absorbs new information.

This research proposes a novel model to compute a hedge ratio. Our model is based on EWMA. Although earlier studies look into a few volatility methods. For instance, a study that concentrates on EWMA-based methodologies finds evidence for the their general reliability in predicting cryptocurrency volatility (Liu et al., 2020). Another study that uses extremely complex GAS models with an assumption of a fat-tailed distribution finds that more complex models do not outperform the simpler ones (Catania & Grassi, 2022).

Table 1.
Summarized Literature

Author(s)	Method(s)	Period	Analysis	Main findings
Jalkh, Bouri, Vo, & Dutta (2020)	Corrected DCC-GARCH	May 2007 – June 2019	in-sample	OVX is a hedge for travel stocks.
Batten, Kinateder, Szilagyi, & Wagner (2019)	DCC-GARCH	January 1990 – December 2017	in-sample	The hedge ratios of the oil-stock portfolio are not constant and more expensive after the global financial crisis, and VIX significantly affects hedged portfolio returns.
Antonakakis, Cunado, Filis, Gabauer, & Perez de Gracia (2018)	DCC-GARCH	June 2001 – February 2016	in-sample	The hedge ratios are not constant, and the values are higher during the global financial crisis.
Antonakakis, Cunado, Filis, Gabauer, & de Gracia (2020)	DCC-*t*-Copula	March 2011 – December 2018	in-sample	The most effective hedging method for OVX is the hedge ratio approach instead of the optimal weights model.
Narayan & Sharma (2016)	Bivariate GARCH	January 2008 – July 2010	in-sample and out-of-sample	Traders should take a short position in the S&P futures market to reduce the risk of the Chinese spot market.
Hamma, Ghorbel, & Jarboui (2021)	DCC-GARCH, ADCC-GARCH, Flexible DCC-GARCH	December 2007 – September 2016	out-of-sample	VISTOXX is the best asset to hedge Islamic equity, and DCC-GARCH is the best-fit method to compute the hedge ratio.
Naeem et al. (2023)	ADCC-GARCH	December 2008 – December 2020	in-sample	The stock markets and Sukuk have a positive and significant correlation that suggests non-hedge characteristics.
Naeem, Mbarki, Alharthi, Omri, & Shahzad (2021)	ADCC-GARCH	May 2013 – August 2020	out-of-sample	A long position in green bonds protects a long position in USD.
Yousaf, Ali, Naveed, & Adeel (2021)	DCC-GARCH	2000 – 2018	in-sample	More oil assets are needed to reduce equity risk during the crisis.
Bandhu Majumder (2022)	VAR-BEKK, VAR-asymmetric BEKK model, VAR-DCC, and VAR-ADCC	October 2013 – December 2020	in-sample	Gold is not a strong hedge for the Indian stock market.

Table 1.
Summarized Literature (Continued)

Author(s)	Method(s)	Period	Analysis	Main findings
Zghal, Melki, & Ghorbel (2022)	DCC-GARCH, ADCC, and GO-GARCH	December 2010 – September 2020	out-of-sample	The ADCC model is the best hedging method for Latin American-commodities portfolio. GO-GARCH is the best-suited formula for the Eastern Europe-commodities context.
Jeribi & Fakhfekh (2021)	FIEGARCH-EVT-Copula, Ordinary Least Square	January 2016 – December 2019	in-sample	Cryptocurrencies are not strong hedges for equity markets.
Ahmad, Sadorsky, & Sharma (2018)	DCC-GARCH and GO-GARCH	March 2008 – October 2017	out-of-sample	VIX is a strong hedge for clean energy equity
Nekhili & Sultan (2022)	GARCH, Wavelet, OLS	April 2013 – Jan 2020	in-sample and out-of-sample	Equity indices can reduce the variance of the Bitcoin in the portfolio.
Izadi & Hassan (2018)	DCC-GARCH	Jan 2000 – October 2014	in-sample and out-of-sample	Gold futures can reduce equity risk.

III. METHODOLOGY

3.1. Data

The data are from *Bloomberg*: SP Funds Dow Jones Global Sukuk ETF (ticker name: SPSK:US), Wahed FTSE USA Shariah ETF (HLAL:US), and iShares MSCI EM Islamic UCITS ETF (ISDE: LN). SPSK includes investment-grade Sukuk denominated in US dollar. The data are daily spanning from Jan 02, 2020 to Oct 25, 2023. The starting date of the sample is dictated by availability of Sukuk data, where the Sukuk ETF was launched at the end of December 2019. All prices are denominated in US dollars. Additionally, the returns of these assets are calculated by taking the first difference of natural log of their respective prices

This study selects exchange-traded funds (ETF) for two main reasons. First, the optimal hedge ratios of Islamic ETFs have not been fully explored (see Table 1). Second, in the ETF markets, where market makers search for the most affordable solutions to lower the risk of their holdings, hedging is a very important tool.

3.2. Out-of-Sample Settings

This study employs an out-of-sample testing strategy to reduce data mining bias. A "training" group of data and a "validation" data set are created from the entire historical sample. The validation group of data is referred to as the "out-of-sample" data, whereas "in-sample" data refers to the training pool. The main issue is the lack of instructions on selecting the boundary that divides in-sample and out-of-sample groups. The problem arises when the in-sample period is brief, which renders data mining bias significant. Conversely, a short out-of-sample section reduces the statistical significance of out-of-sample performance tests. Hence, this study uses two split points (75% in-sample period and 50% in-sample period). Moreover, since the market dynamics have always changed, this research uses a rolling-window approach.

3.3. Naïve and Linear Regression (OLS)

The naïve approach assumes that the hedge ratio is 1. The OLS hedge ratio (conventional hedging) uses the following model (Kroner & Sultan, 1993):

$$s_t = \alpha + \beta f_t + \varepsilon_t \tag{1}$$

s_t is Islamic equity, while f_t is Sukuk. The computation of time-varying β (hedge ratio) uses a rolling-window framework.

3.4. EWMA (Exponentially Weighted Moving Average)

EWMA model puts more weight on the more recent observations. As extreme returns move further into the past when the data window moves, they become less important in the average. EWMA for the variance estimate at time t of returns is

$$\check{\sigma}_t^2 = (1-\lambda)r_{t-1}^2 + \lambda\check{\sigma}_{t-1}^2 \tag{2}$$

λ is the smoothing constant.

3.5. EWMA with Volatility Response

This model is the main contribution of this paper. Carol Alexander & Dakos (2023) develop an asymmetric volatility response (η) to the original EWMA model:

$$\check{\sigma}_t^2 = (1-\lambda)(r_{t-1} - \eta)^2 + \lambda\check{\sigma}_{t-1}^2 \tag{3}$$

This study uses 0.001 and 0.002 for η parameters. Carol Alexander & Dakos (2023) suggest 0.01 to 0.03 for the η parameters since they use cryptocurrencies, which have higher volatilities than equity. Further, regulators might consider having a decay value (λ) below 0.90 unacceptable, and the asymmetry values (η) must not be large to obscure the effect of realized returns on volatility (Carol Alexander & Dakos, 2023).

3.6. Dynamic Equicorrelation (DECO-GARCH)

The conditional covariance matrix (H_t) of DECO-GARCH is computed by the following procedures:

$$H_t = D_t^{1/2} R_t D_t^{1/2} \tag{4}$$

and D_t is the diagonal matrix of conditional variance. The correlation matrix is

$$R_t^{DCC} = (Q_t^*)^{-1/2} Q_t (Q_t^*)^{-1/2} \tag{5}$$

$$Q_t = (1 - \varphi - \xi)K + \varphi\eta_{t-1}\eta'_{t-1} + \xi Q_{t-1} \tag{6}$$

R_t^{DCC} is the correlation matrix based on the Dynamic Conditional Correlation (DCC)-GARCH, φ is a non-negative scalar, η_t is the standardized residuals, and K is the unconditional covariance matrix of η_t. The conditional correlation of the equicorrelation framework (Engle & Kelly, 2012) is

$$R_t^{deco} = (1 - \rho_t)I_n + \rho_t J_n \tag{7}$$

I_n is the $\boldsymbol{n}$-dimensional identity matrix, in this case, three assets: $\boldsymbol{n} = 3$, and J_n 3 x 3 matrix of ones. The DECO model produces ρ_t. It is based on the average DCC correlations.

$$\rho_t^{deco} = \frac{1}{n(n-1)}\left(J'_n R_t^{deco} J_n - n\right) = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}} \tag{8}$$

This study also estimates the ADCC-GARCH (Cappiello, Engle, & Sheppard, 2006) and GO-GARCH (Van der Weide, 2002).

3.7. Hedge Ratio

A hedge ratio (HR) determines the costs of hedging a long position in Islamic equity (denoted by *s*) with a short position in Sukuk (represented by *f*). Then, the hedge ratio is

$$\hat{\beta}_{sf,t} = \frac{h_{sf,t}}{h_{ff,t}} \tag{9}$$

$h_{sf,t}$ shows the conditional variance of Sukuk *f* and Islamic equity *s*, and $h_{ff,t}$ the conditional variance of *f*.

3.8. Wavelet Analysis

This study transforms the raw results using a wavelet algorithm to handle the heterogeneous timing preferences (Qureshi, Aftab, Bouri, & Saeed, 2020). One benefit of the wavelet analysis is the ability to structure a hedger's multi-period decisions due to the clientele effect (Kamara, Korajczyk, Lou, & Sadka, 2016). Wavelets are nonlinear and square-integrable functions, as shown by:

$$\psi_{u,s} = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right) \tag{10}$$

u is a location parameter, s is a scale parameter, and $\frac{1}{\sqrt{s}}$ is a normalization factor. The continuous wavelet transform is:

$$W_{u,s}(t) = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right) dt \tag{11}$$

This procedure decomposes and reconstructs a time series $x(t)$:

$$x(t) = \frac{1}{C_\psi}\int_0^{\infty} \left[\int_{-\infty}^{+\infty} W_s(u,s)\psi_{u,s}(t)du\right]\frac{ds}{s^2}, s > 0 \tag{12}$$

There are two methods to implement a wavelet framework: *the continuous wavelet transform* (CWT) and *the discrete wavelet transform* (DWT). This research applies *the maximal overlap discrete wavelet transform* (MODWT). The MODWT offers some advantages (Bouri, Shahzad, Roubaud, Kristoufek, & Lucey, 2020). The MODWT can manage any sample size to begin with, which makes it better suited for determining wavelet correlations. Hence, Eq. (1) can be changed to the following formula:

$$\Delta s_t = s_{j,t}^s + D_{j,t}^s + D_{j-1,t}^s + \cdots + D_{1,t}^s \; \Delta f_t = f_{j,t}^f + D_{j,t}^f + D_{j-1,t}^f + \cdots + D_{1,t}^f \tag{13}$$

The wavelet framework modifies Eq. 9:

$$\hat{\beta}_{sf,t} = \frac{h_{sf,t,j}}{h_{ff,t,j}} \tag{14}$$

3.9. Hedging Evaluation

This study also evaluates whether the hedge portfolio provides economic significance for investors. First, this study calculates the variance reduction, which is the variance difference between an unhedged portfolio and a hedged portfolio (Ederington, 1979). The variance of the hedged portfolio is:

$$var(\Omega_{t-1}) = var\left(\left(Y_t^{Islamic\ equity} - \hat{\beta}_{sf,t}Y_t^{Sukuk}|\Omega_{t-1}\right)\right) \tag{15}$$

$Y_t^{Islamic\ equity}$ is the return of Islamic equity, while Y_t^{Sukuk} is the return of Sukuk and the variance of the unhedged portfolio ($\mathbb{up}$) is obtained by:

$$var(\text{'}\Omega_{t-1}) = var\ \left(\left(Y_t^{Islamic\ equity} | \text{'}\Omega_{t-1}\right)\right) \tag{16}$$

Secondly, this study uses a mean-variance utility function (Kroner & Sultan, 1993):

$$\dot{\mathrm{E}}[\bar{\mathrm{U}}(hp_t | \text{'}\Omega_{t-1})] = \dot{\mathrm{E}}(\text{'}\Omega_{t-1}) - \gamma var(\text{'}\Omega_{t-1}) - TC \tag{17}$$

where γ indicates the investor's risk aversion value, four, and TC is the transaction cost of 0.05%. The expected hedged portfolio return, $\dot{\mathrm{E}}(\Omega_{t-1})$, is zero.

Lastly, this study also considers the effects of skewness and kurtosis on hedging effectiveness. The large kurtosis would suggest that the hedge is sometimes ineffective. Furthermore, if the returns from the hedged portfolio are negatively skewed, the hedged position will occasionally lose money rather than gain it. Based on these assumptions, the certainty equivalent utility function (CE) is:

$$ce = \mu - \frac{\sigma^2}{2v} + \frac{\varphi}{6v^2} - \frac{\text{ƙ}}{24v^3} \tag{18}$$

φ and σ are the skewness and the standard deviation of the hedge portfolio returns, respectively. μ and ƙ are the mean and kurtosis of the hedge portfolio returns, respectively. The value of v is 0.1 (C. Alexander & Barbosa, 2008). We use CE as our main performance evaluation.

3.10. Minimum Confidence Set (MCS)

According to Hansen, Lunde, & Nason (2011), the existing data may sometimes lack sufficient information to produce a single model that significantly outperforms other models. Therefore, it is possible to acquire a reduced collection of models, known as a Minimum Confidence Set (MCS), which includes the optimal model with a specified level of confidence.

IV. RESULTS AND ANALYSIS

4.1. Preliminary Analysis

This section reports the basic statistics of the data and GARCH parameters. Table 2 reveals that the kurtosis values are greater than 3, showing a leptokurtic distribution. The results of the Jarque-Bera, D'Agostino, and Anscombe-Glynn tests indicate that the null hypothesis of no normality, excess skewness, and kurtosis could not be supported. The assumption of a zero mean applies to all assets, with a mean close to zero. As expected, the daily returns of Islamic equity are more volatile than Sukuk returns according to the standard deviation. In addition, Sukuk are positively correlated with Islamic equity.

Further, Figure 1 illustrates the existence of volatility clustering. In addition, the news impact curves offer a logical representation of the volatility response. The plots demonstrate that negative log-returns have a more dramatic effect on news impact than positive log-returns, except for Sukuk, which are more symmetric.

Table 2.
Descriptive Statistics of Daily Log-Returns, Jan 02, 2020 – Oct 25, 2023

	Mean (%)	Standard Deviation (%)	Skewness	Kurtosis	Min (%)	Max (%)	Jarque-Bera	Correlation with Sukuk
Sukuk	-0.004	0.323	-0.104*** (10.261)	7.680 (-1.311)	-1.640	2.000	859***	1.000
FTSE USA	0.036	1.462	-0.799*** (12.827)	11.918*** (-8.941)	-10.891	8.052	3215***	0.208
Emerging markets	-0.017	1.408	-0.975*** (13.636)	14.059*** (-10.461)	-13.233	8.404	4939***	0.195

Notes: The numbers in parentheses are the statistics of the D'Agostino and Anscombe-Glynn for testing the null hypothesis of no excess kurtosis and skewness. *** shows statistical significance at 0.01 level.

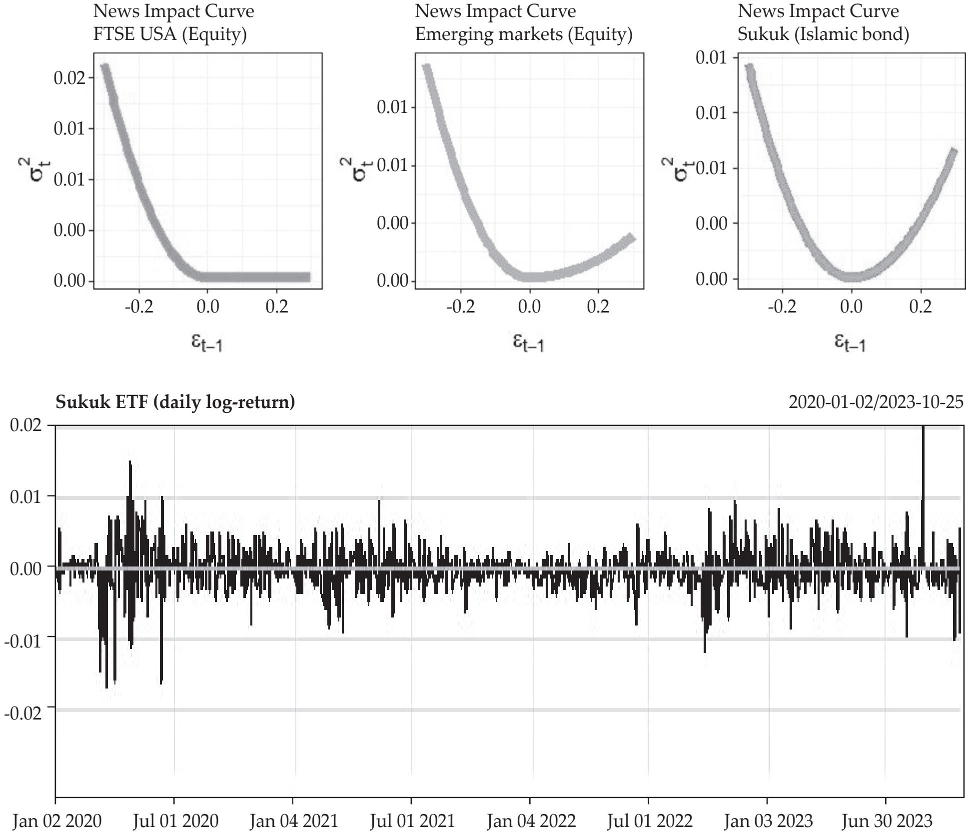
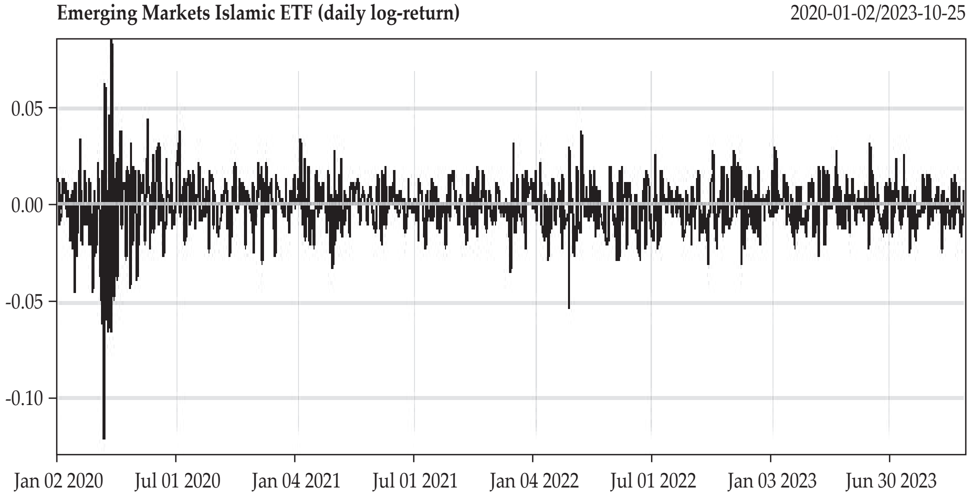


Figure 1.
Daily Log-Return and News Impact Curves, Jan 02, 2020-Oct 25, 2023



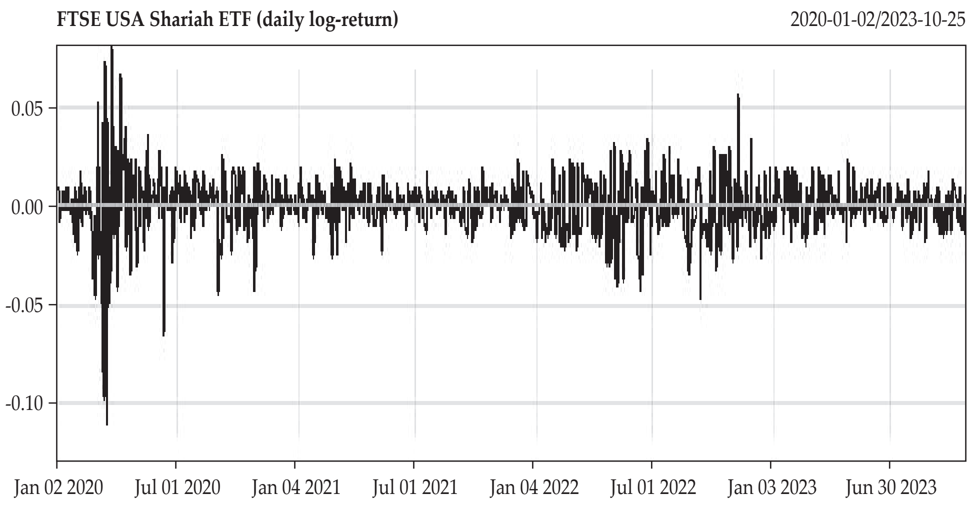


Figure 1.
Daily Log-Return and News Impact Curves, Jan 02, 2020-Oct 25, 2023 (Continued)

4.2. Out-of-Sample Performance - Raw Returns

This section focuses on the performance of hedged portfolios using raw returns. Figure 2 shows the dynamic hedge ratio (HR). Only an analysis of 50% in-sample data is presented to save space. The negative value of HR indicates that traders should take the same position (either short or long) for both assets. In contrast, the positive value of HR indicates that inverse positions (long and short) are required. In this case, all models produce positive median values of HR applying both 50 % in-sample and 75 % in-sample data.

Table 3 presents the descriptive statistics of the hedged portfolio returns. Negative skewness indicates that hedgers may experience numerous minor gains and a few significant losses. A high kurtosis indicates a high level of risk, and hedging might fail catastrophically. For instance, hedged portfolio returns of Islamic equity in emerging markets and Sukuk, generated by the naïve method, have the lowest kurtosis (75% in-sample period). In addition, hedged portfolio returns of Islamic equity in the US and Sukuk, based on ADCC-GARCH, have the lowest kurtosis (50% in-sample period).

In addition, Table 4 presents the hedging performance based on three evaluation models (raw returns). The OLS method has higher Ederington values (50% in-sample data). Further, GARCH, OLS, and naive models have higher certainty equivalent values than the EWMA-based models indicating that EWMA-based models are riskier for the portfolio consisting FTSE USA and Sukuk (50% in-sample period).

Further, Table 5 shows the ρ-value minimum confidence set (raw returns). Note that this study uses certainty equivalence as the main evaluation criterion. The table shows that the ρ-values of the minimum confidence set are above 0.10, implying that all models have equal performance.

Table 3.
Basic Statistics of Hedged Portfolio - Raw Returns

	75% in-sample period				50% in-sample period			
	Mean (%)	**Volatility (%)**	**Skewness**	**Kurtosis**	**Mean (%)**	**Volatility (%)**	**Skewness**	**Kurtosis**
DECO-GARCH								
FTSE USA/Sukuk	0.041	1.009	0.504	5.373	0.020	1.271	0.027	3.681
Emerging markets/Sukuk	-0.036	1.040	0.110	3.219	-0.047	1.140	-0.071	3.909
ADCC-GARCH								
FTSE USA/Sukuk	0.041	1.009	0.504	5.373	0.019	1.272	0.028	3.675
Emerging markets/Sukuk	-0.034	1.042	0.114	3.219	-0.048	1.138	-0.078	3.907
GO-GARCH								
FTSE USA/Sukuk	0.033	1.045	0.332	5.012	0.023	1.288	-0.026	3.771
Emerging markets/Sukuk	-0.019	1.044	0.149	3.306	-0.044	1.151	-0.065	3.870
EWMA (94%)								
FTSE USA/Sukuk	0.046	1.022	0.421	5.200	0.019	1.280	0.023	3.716
Emerging markets/Sukuk	-0.033	1.047	0.126	3.256	-0.043	1.151	-0.048	3.842
EWMA (94%, 0.1%)								
FTSE USA/Sukuk	0.046	1.021	0.411	5.262	0.009	1.273	0.007	3.810
Emerging markets/Sukuk	-0.032	1.045	0.121	3.265	-0.052	1.146	-0.050	3.819
EWMA (94%, 0.2%)								
FTSE USA/Sukuk	0.047	1.017	0.433	5.275	0.001	1.273	-0.010	3.901
Emerging markets/Sukuk	-0.030	1.040	0.122	3.290	-0.058	1.143	-0.053	3.812
EWMA (92.5%)								
FTSE USA/Sukuk	0.046	1.026	0.394	5.154	0.020	1.284	0.009	3.715
Emerging markets/Sukuk	-0.035	1.051	0.120	3.285	-0.043	1.156	-0.047	3.824
EWMA (92.5%, 0.1%)								
FTSE USA/Sukuk	0.046	1.026	0.380	5.223	0.009	1.276	-0.007	3.807
Emerging markets/Sukuk	-0.033	1.049	0.112	3.297	-0.053	1.149	-0.050	3.811

Table 3.
Basic Statistics of Hedged Portfolio - Raw Returns (Continued)

	75% in-sample period				50% in-sample period			
	Mean (%)	**Volatility (%)**	**Skewness**	**Kurtosis**	**Mean (%)**	**Volatility (%)**	**Skewness**	**Kurtosis**
EWMA (92.5%, 0.2%)								
FTSE USA/Sukuk	0.047	1.021	0.406	5.242	0.001	1.274	-0.015	3.895
Emerging markets/Sukuk	-0.031	1.042	0.112	3.318	-0.058	1.145	-0.055	3.809
OLS								
FTSE USA/Sukuk	0.038	1.052	0.335	4.927	0.016	1.270	0.006	3.754
Emerging markets/Sukuk	-0.031	1.036	0.116	3.195	-0.055	1.137	-0.067	3.852
NAÏVE								
FTSE USA/Sukuk	0.053	1.028	0.500	5.291	0.006	1.278	-0.014	3.804
Emerging markets/Sukuk	-0.029	1.046	0.108	3.168	-0.050	1.138	-0.073	3.879

Notes: This table reports the descriptive statistics of hedged portfolio returns. 50 % in-sample means that the in-sample data are from Jan 02, 2020, to Nov 24, 2021, and the out-of-sample period is from Nov 26, 2021, to Oct 25, 2023. 75% in-sample means that the in-sample data are from Jan 02, 2020, to Nov 08, 2022, and the out-of-sample period is from Nov 09, 2022, to Oct 25, 2023. The mean values show average daily returns. Negative skewness indicates that hedgers may experience numerous minor gains and a few significant losses. A high kurtosis indicates a high level of risk, and hedging might fail catastrophically.

Table 4.
Hedging Performance - Raw Returns

	Ederington		Mean-Variance Utility		Certainty Equivalent	
75% in-sample period						
	FTSE USA/ Sukuk	**Emerging markets/ Sukuk**	**FTSE USA/ Sukuk**	**Emerging markets/ Sukuk**	**FTSE USA/ Sukuk**	**Emerging markets/ Sukuk**
DECO-GARCH	0.004	0.023	-0.096	-0.102	-212.477	-132.285
ADCC-GARCH	0.004	0.020	-0.095	-0.102	-212.857	-132.248
GO-GARCH	-0.068	0.018	-0.103	-0.103	-203.850	-137.556
EWMA (94%)	-0.020	0.011	-0.098	-0.104	209.642	-133.590
EWMA (94%, 0.1%)	-0.020	0.015	-0.098	-0.104	-212.398	-134.033
EWMA (94%, 0.2%)	-0.011	0.025	-0.097	-0.104	-212.591	-135.069
EWMA (92.5%)	-0.029	0.003	-0.099	-0.103	-208.180	-134.914
EWMA (92.5%, 0.1%)	-0.029	0.007	-0.099	-0.103	-211.328	-135.517
EWMA (92.5%, 0.2%)	-0.018	0.020	-0.098	-0.102	-211.648	-136.383
OLS	-0.081	0.032	-0.104	-0.103	-199.697	-131.231
NAÏVE	-0.034	0.012	-0.100	-0.104	212.155	-130.209
50% in-sample period						
	Ederington		**Mean-Variance Utility**		**Certainty Equivalent**	
	FTSE USA/ Sukuk	**Emerging markets/ Sukuk**	**FTSE USA/ Sukuk**	**Emerging markets/ Sukuk**	**FTSE USA/ Sukuk**	**Emerging markets/ Sukuk**
DECO-GARCH	0.036	0.031	-0.304	-0.245	-152.931	-164.096
ADCC-GARCH	0.034	0.035	-0.304	-0.244	-152.686	-164.081
GO-GARCH	0.009	0.013	-0.312	-0.249	-157.577	-162.347
EWMA (94%)	0.022	0.012	-0.308	-0.249	-154.460	-160.902
EWMA (94%, 0.1%)	0.032	0.022	-0.305	-0.247	-158.643	-160.003
EWMA (94%, 0.2%)	0.032	0.026	-0.305	-0.246	-162.760	-159.754
EWMA (92.5%)	0.015	0.004	-0.310	-0.251	-154.646	-160.129
EWMA (92.5%, 0.1%)	0.028	0.016	-0.306	-0.248	-158.644	-159.666
EWMA (92.5%, 0.2%)	0.030	0.022	-0.306	-0.247	-162.561	-159.645
OLS	0.037	0.036	-0.303	-0.243	-156.331	-161.638
NAÏVE	0.025	0.034	-0.307	-0.244	-158.741	-162.849

Notes: The table reports the hedging performance of the hedged portfolio returns. The Ederington value is the difference in variance between hedged and un-hedged portfolios. The mean-variance utility considers the investor's risk aversion value. To account for the investor's resistance to negative skewness and positive excess kurtosis, the certainty equivalent is derived from the return data and an exponential utility function. Higher values are better than lower values.

Table 5.
ρ-values of Minimum Confidence Set - Raw Returns

	50% in-sample period		75% in-sample period	
	FTSE USA/ Sukuk	**Emerging markets/Sukuk**	**FTSE USA/ Sukuk**	**Emerging markets/Sukuk**
DECO-GARCH	0.630	0.990	0.170	0.598
ADCC-GARCH	0.681	0.917	0.160	0.606
GO-GARCH	0.422	0.969	0.309	0.461
EWMA (94%)	0.563	0.233	0.251	0.596
EWMA (94%, 0.1%)	0.583	0.349	0.223	0.598
EWMA (94%, 0.2%)	0.594	0.353	0.274	0.567
EWMA (92.5%)	0.531	0.137	0.113	0.259
EWMA (92.5%, 0.1%)	0.621	0.173	0.212	0.560
EWMA (92.5%, 0.2%)	0.585	0.215	0.311	0.515
OLS	1.000	1.000	1.000	0.222
NAÏVE	0.897	0.247	0.890	1.000

Notes: This table reports the ρ-values of the minimum confidence set (raw returns). The MCS is used to statistically determine the best-fitting model(s) based on certainty equivalent evaluation. Models that have ρ-values above 0.1 outperform the models that have ρ-values below 0.1.

4.3. Out-of-Sample Performance - Wavelet-Transformed Returns

The previous results do not take into consideration heterogeneous timing preferences. Hence, this section focuses on wavelet-based hedge ratios. Figure 3 shows the wavelet-based hedge ratios. Only an analysis of 50% in-sample data is presented to save space. All models produce positive median values of HR applying both 50 % in-sample period and 75 % in-sample period. It means that inverse positions (long and short) are required. Taking a long position in Islamic equity must be followed by a short position in Sukuk.

Further, Table 7 presents the hedging performance based on the wavelet transformed returns (2-4 days). The proposed models have higher certainty equivalent values in 50% in-sample data and 75% in-sample data. These results indicate that the models are less risky than GARCH, OLS, and naïve models.

Moreover, Table 8 shows that the proposed models are the best-fitting models. We also analyze the hedging performance based on the wavelet transformed returns (68-128 days). Table 9 indicates that the proposed models have higher certainty equivalent values in 50% in-sample data and 75% in-sample data.

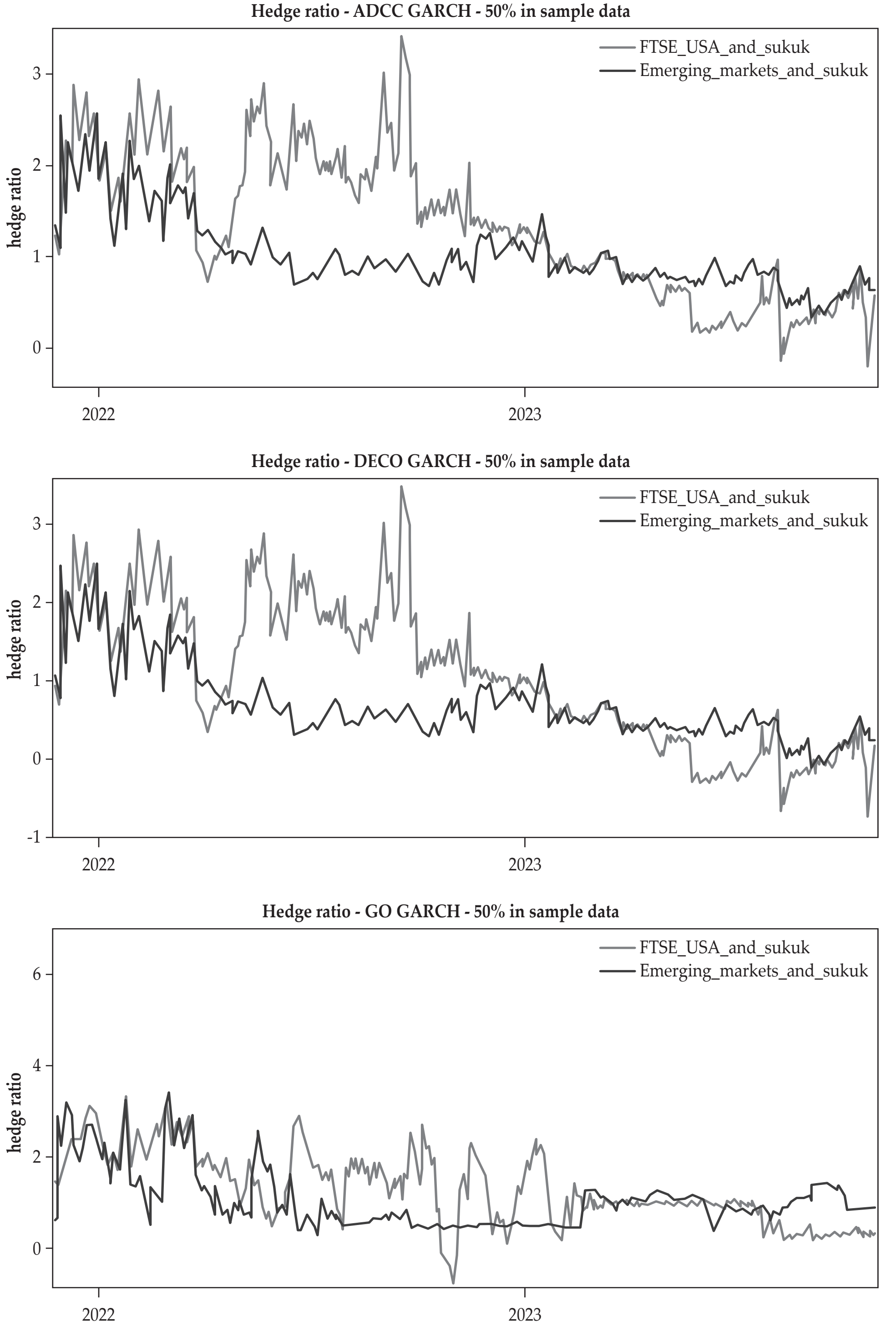


Figure 2.
Dynamic Hedge Ratios - Raw Returns

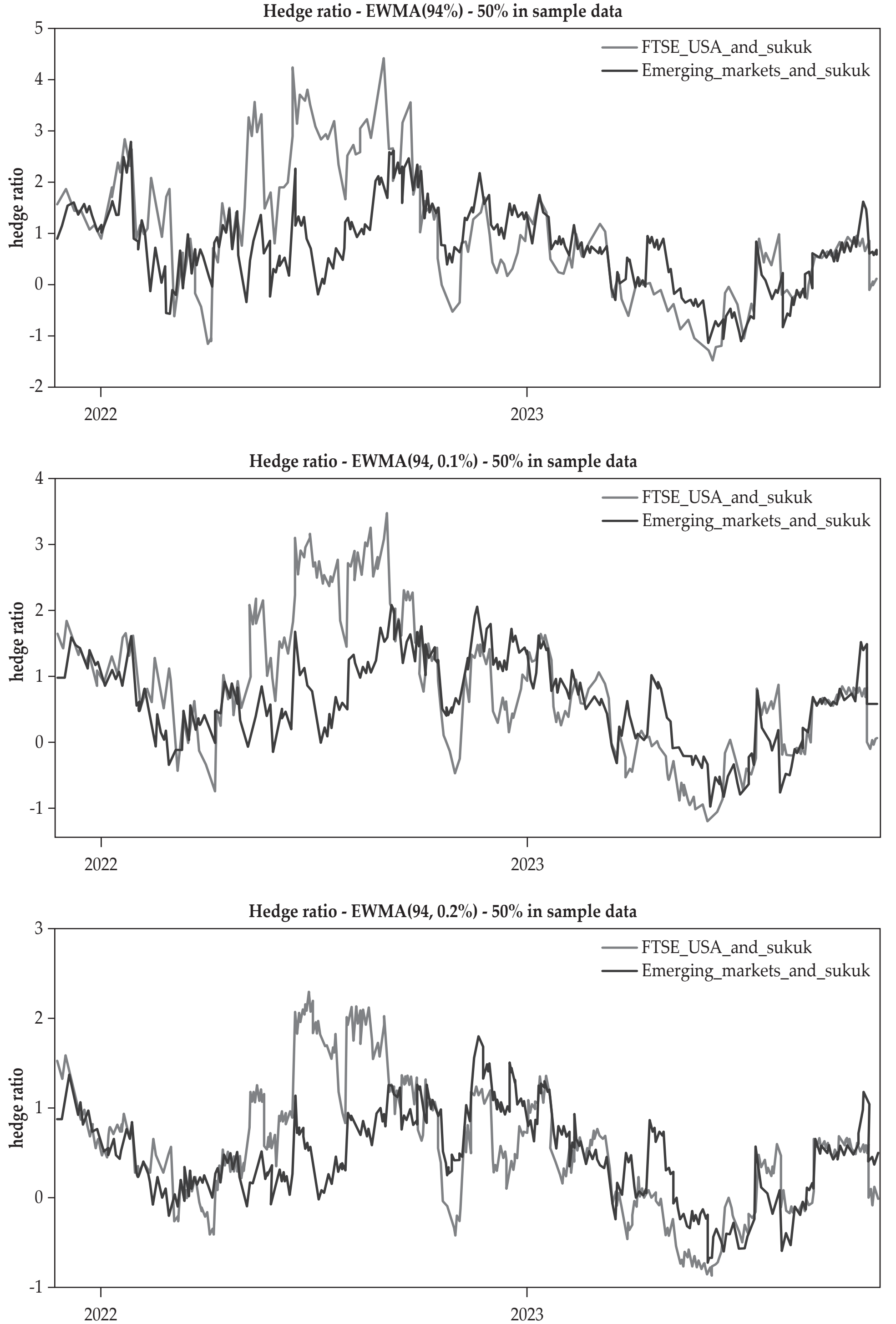


Figure 2.
Dynamic Hedge Ratios - Raw Returns (Continued)

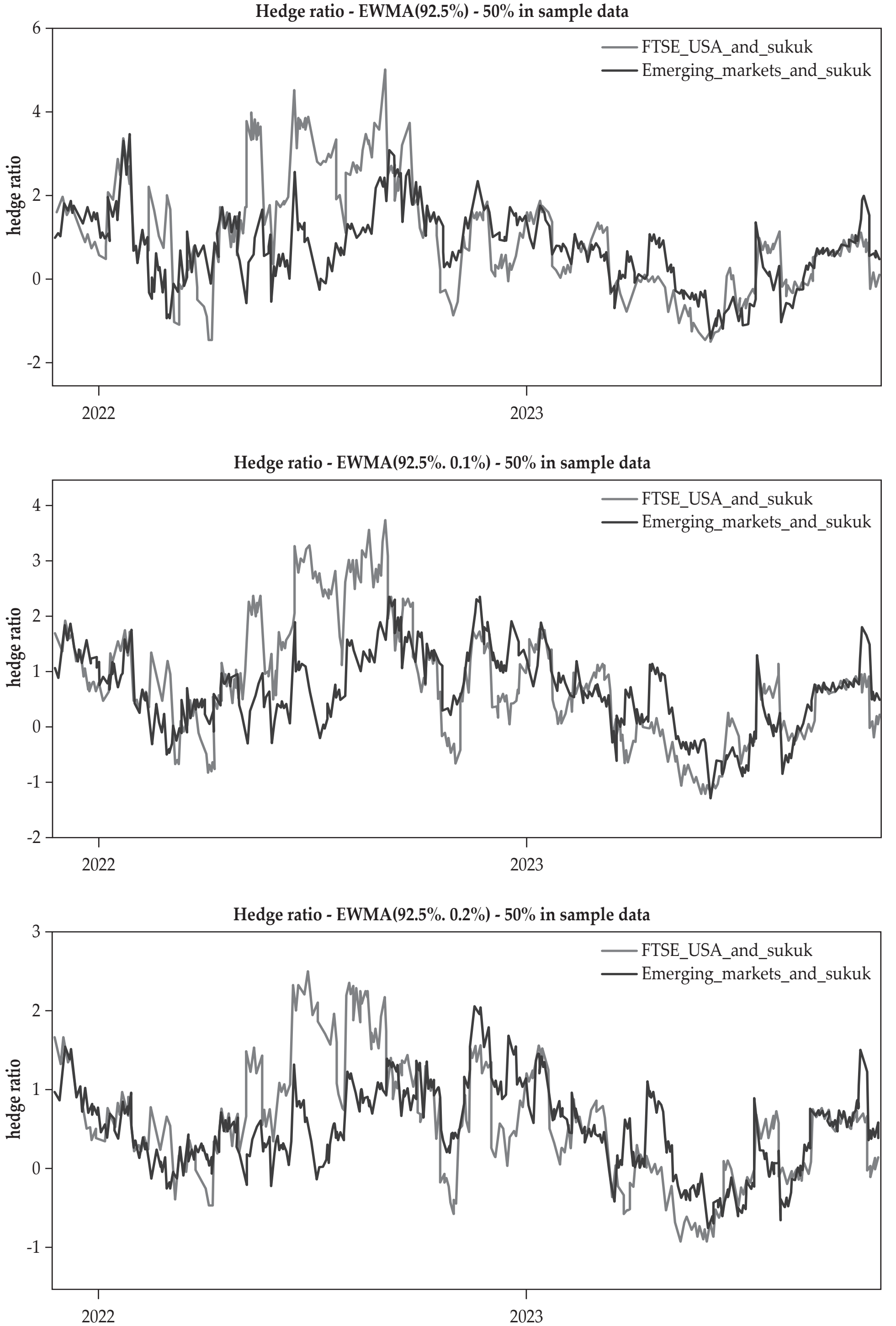
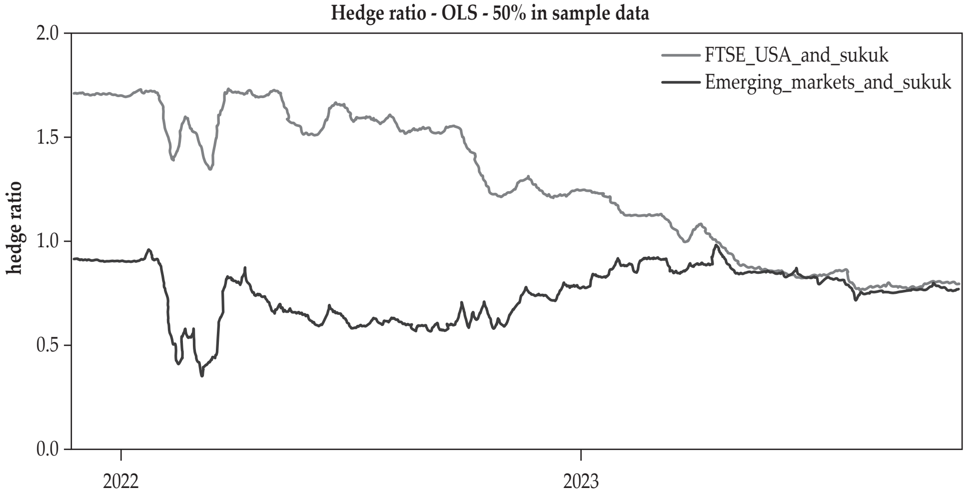


Figure 2.
Dynamic Hedge Ratios - Raw Returns (Continued)



Notes: These figures show the out-of-sample estimates of the one-period-ahead hedge ratios. All methods produce positive median values of hedge ratios, indicating that inverse positions (long and short) are required. The red lines represent Islamic equity from USA and Sukuk portfolio while the blue lines indicate Islamic equity from emerging markets and Sukuk portfolio.

Figure 2.
Dynamic Hedge Ratios - Raw Returns (Continued)

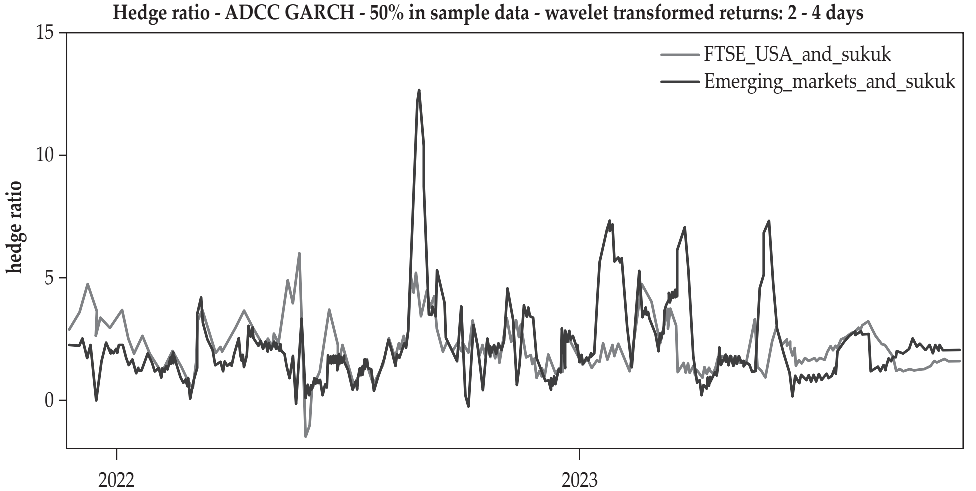


Figure 3.
Dynamic Hedge Ratios - Wavelet Transformed Returns (2-4 days)

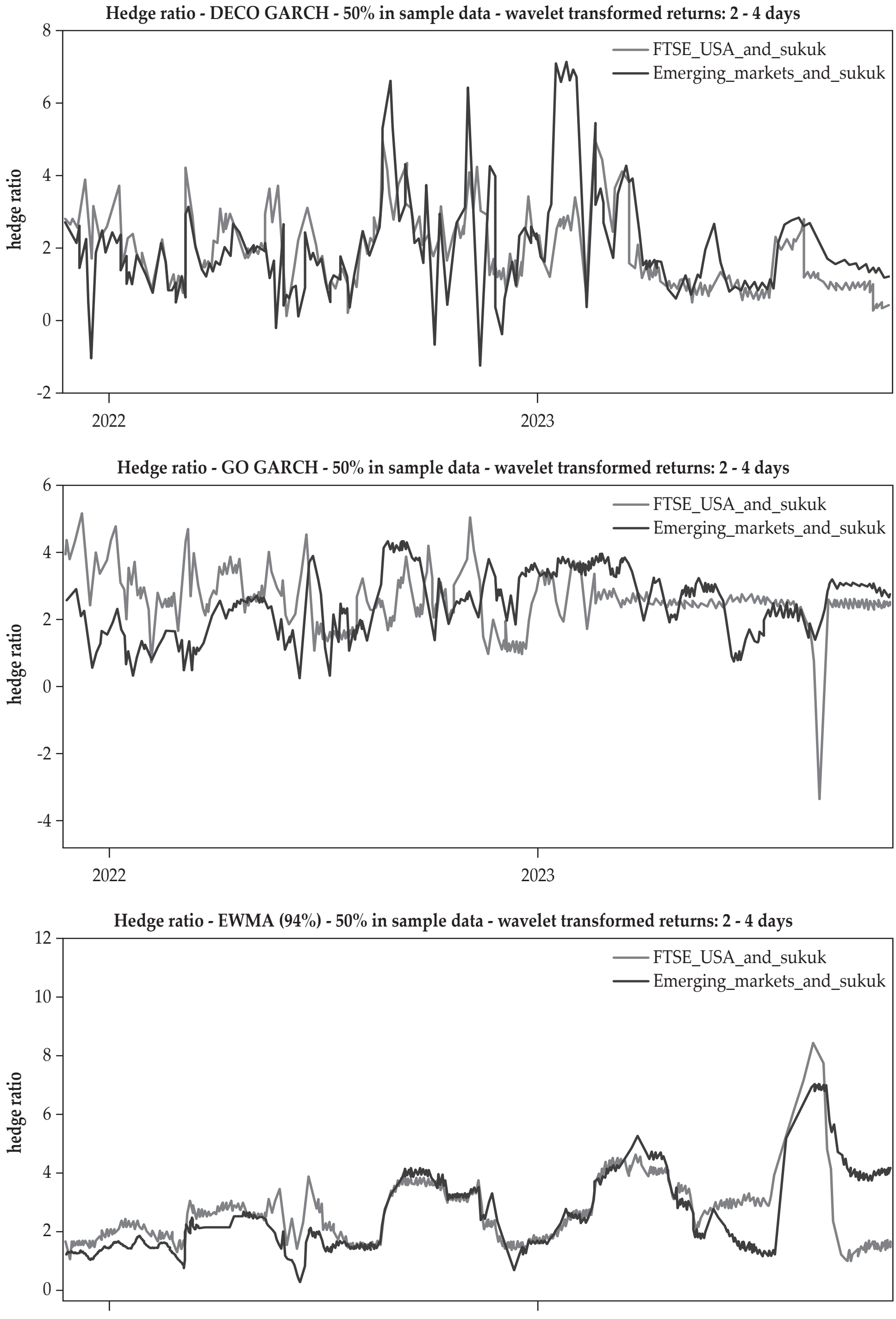
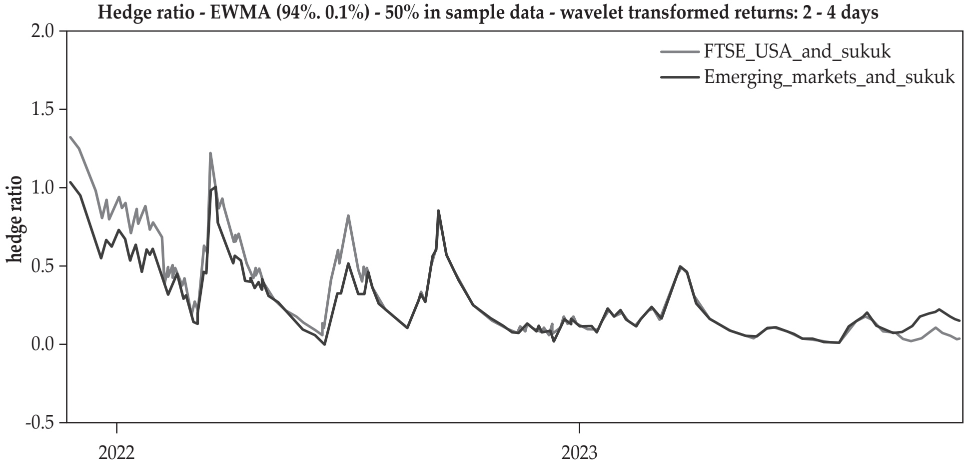
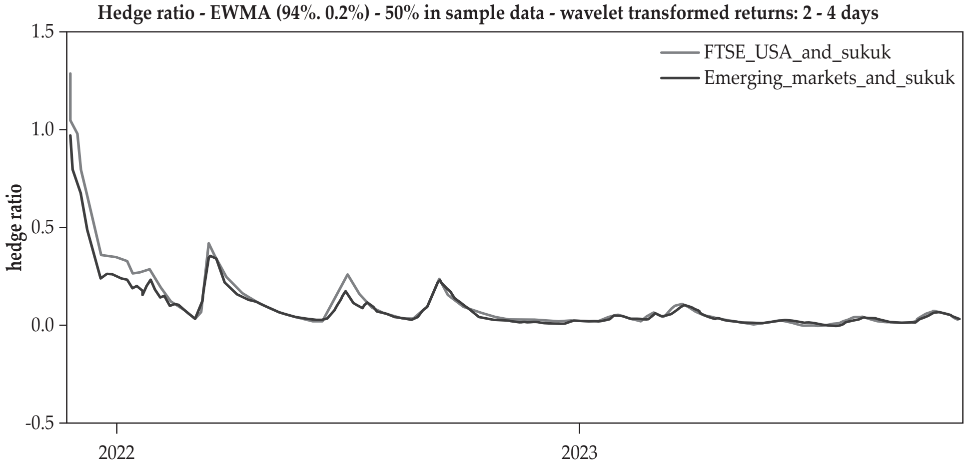


Figure 3.
Dynamic Hedge Ratios - Wavelet Transformed Returns (2-4 days) (Continued)





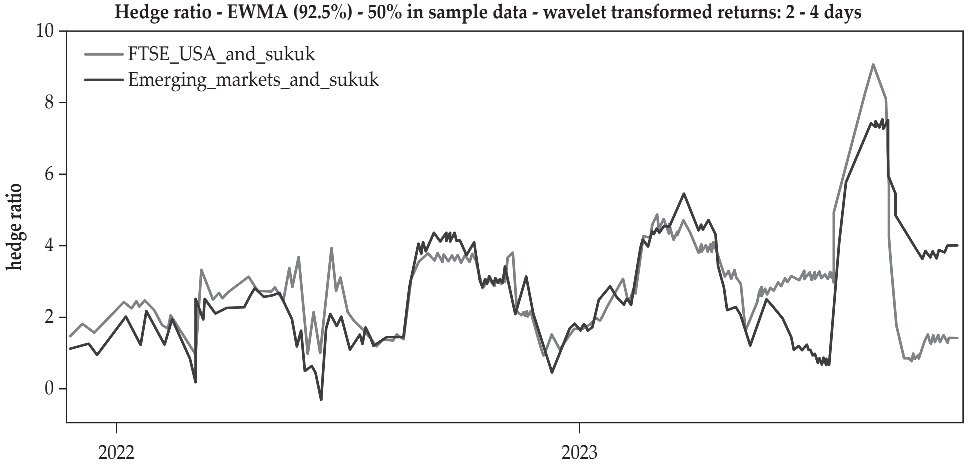
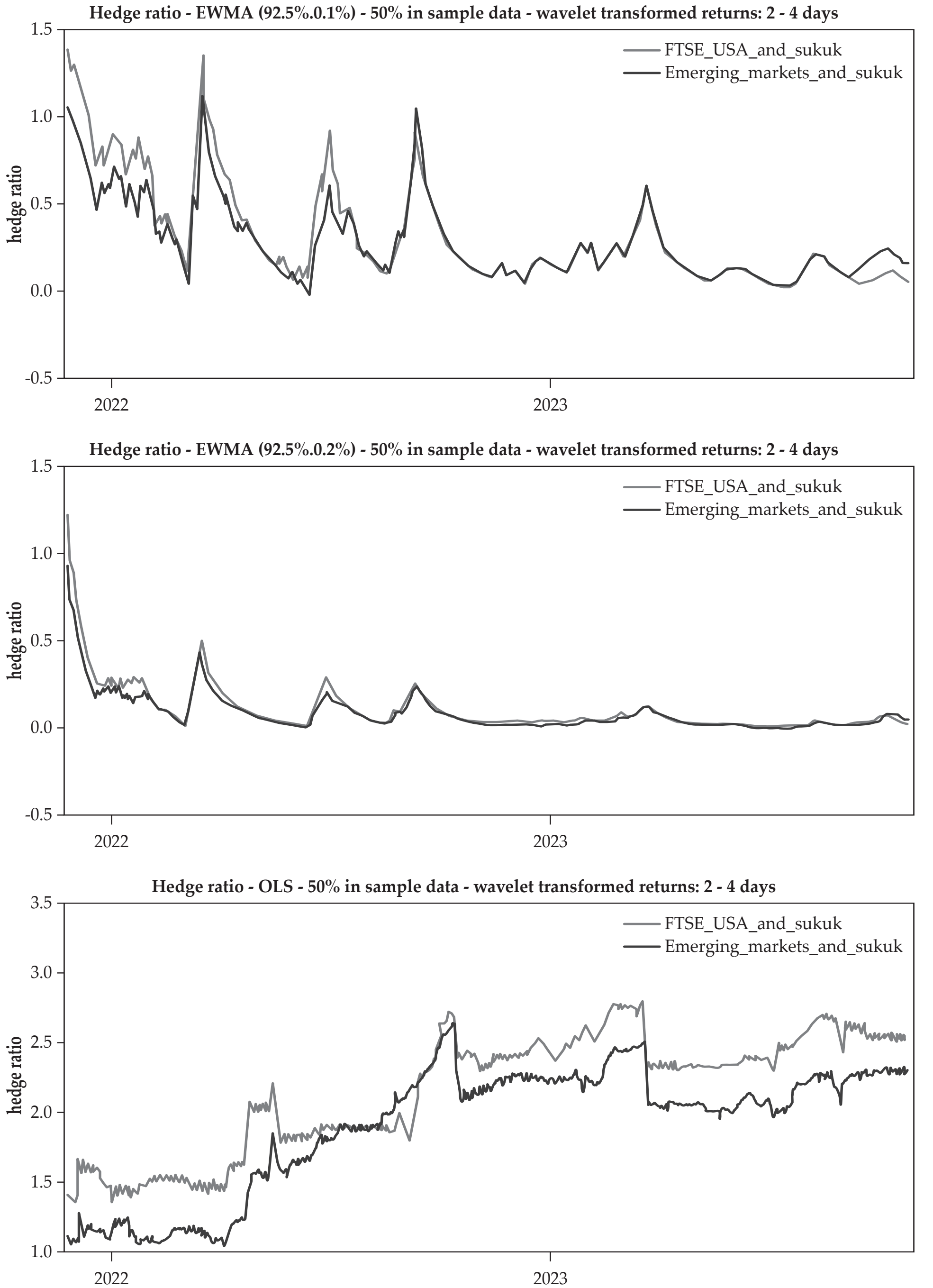


Figure 3.
Dynamic Hedge Ratios - Wavelet Transformed Returns (2-4 days) (Continued)



Notes: These figures show the out-of-sample estimates of the one-period-ahead hedge ratios. All methods produce positive median values of hedge ratios, indicating that inverse positions (long and short) are required. The red lines represent Islamic equity from the US and Sukuk portfolio while the blue lines indicate Islamic equity from emerging markets and Sukuk portfolio.

Figure 3.
Dynamic Hedge Ratios - Wavelet Transformed Returns (2-4 days) (Continued)

Table 6.
Basic Statistics of Hedged Portfolio - Wavelet Transformed Returns (2-4 days)

	75% in-sample period				50% in-sample period			
	Mean (%)	**Volatility (%)**	**Skewness**	**Kurtosis**	**Mean (%)**	**Volatility (%)**	**Skewness**	**Kurtosis**
DECO-GARCH								
FTSE USA/Sukuk	0.013	0.013	0.215	3.304	0.007	0.103	-0.014	8.686
Emerging markets/Sukuk	-0.033	0.049	-0.270	3.703	-0.007	0.102	0.037	6.771
ADCC-GARCH								
FTSE USA/Sukuk	0.011	0.039	-0.096	3.301	0.009	0.102	-0.002	8.664
Emerging markets/Sukuk	-0.012	0.040	0.175	3.026	-0.004	0.105	0.249	6.834
GO-GARCH								
FTSE USA/Sukuk	0.002	0.041	-0.042	3.080	0.009	0.107	0.213	8.351
Emerging markets/Sukuk	-0.010	0.046	0.391	2.614	-0.001	0.106	-0.016	7.338
EWMA (94%)								
FTSE USA/Sukuk	0.011	0.043	-0.156	3.790	0.005	0.106	-0.126	8.013
Emerging markets/Sukuk	-0.001	0.048	-0.228	2.251	-0.001	0.100	-0.078	7.048
EWMA (94%, 0.1%)								
FTSE USA/Sukuk	0.014	0.071	-0.249	2.934	0.007	0.127	-0.131	6.628
Emerging markets/Sukuk	-0.008	0.083	0.408	2.468	-0.004	0.123	0.041	4.848
EWMA (94%, 0.2%)								
FTSE USA/Sukuk	0.015	0.074	-0.245	2.899	0.007	0.132	-0.128	6.717
Emerging markets/Sukuk	-0.008	0.086	0.412	2.508	-0.004	0.127	0.064	4.876
EWMA (92.5%)								
FTSE USA/Sukuk	0.009	0.041	-0.067	3.904	0.005	0.106	-0.096	8.308
Emerging markets/Sukuk	-0.003	-0.003	-0.162	2.448	-0.002	0.100	-0.084	7.449
EWMA (92.5%, 0.1%)								
FTSE USA/Sukuk	0.014	0.071	-0.262	2.919	0.007	0.127	-0.139	6.660
Emerging markets/Sukuk	-0.008	0.083	0.399	2.446	-0.004	0.123	0.026	4.883

Table 6.
Basic Statistics of Hedged Portfolio - Wavelet Transformed Returns (2-4 days) (Continued)

	75% in-sample period				50% in-sample period			
	Mean (%)	Volatility (%)	Skewness	Kurtosis	Mean (%)	Volatility (%)	Skewness	Kurtosis
EWMA (92.5%, 0.2%)								
FTSE USA/Sukuk	0.015	0.015	-0.249	2.886	0.007	0.133	-0.138	6.713
Emerging markets/Sukuk	-0.008	0.086	0.410	2.499	-0.004	0.127	0.057	4.881
OLS								
FTSE USA/Sukuk	0.020	0.020	-0.598	3.419	0.009	0.109	-0.118	7.518
Emerging markets/Sukuk	-0.002	0.064	0.537	2.846	-0.002	0.105	0.070	6.124
NAÏVE								
FTSE USA/Sukuk	0.016	0.060	-0.323	3.231	0.008	0.118	-0.151	6.998
Emerging markets/Sukuk	-0.007	0.071	0.505	2.599	-0.003	0.113	0.108	5.140

Notes: This table reports the descriptive statistics of hedged portfolio returns. 50 % in-sample means that the in-sample data are from Jan 02, 2020, to Nov 24, 2021, and the out-of-sample period is from Nov 26, 2021, to Oct 24, 2023. 75% in-sample means that the in-sample data are from Jan 02, 2020, to Nov 08, 2022, and the out-of-sample period is from Nov 09, 2022, to Oct 24, 2023. The mean values show average daily returns. Negative skewness indicates that hedgers may experience numerous minor gains and a few significant losses. A high kurtosis indicates a high level of risk, and hedging might fail catastrophically.

Table 7.
Hedging Performance - Wavelet Transformed Returns (2-4 days)

	Ederington		Mean-Variance Utility		Certainty Equivalent	
75% in-sample period						
	FTSE USA/ Sukuk	**Emerging markets/ Sukuk**	**FTSE USA/ Sukuk**	**Emerging markets/ Sukuk**	**FTSE USA/ Sukuk**	**Emerging markets/ Sukuk**
DECO-GARCH	0.674	0.674	-0.0006	-0.0007	-134.074	-158.798
ADCC-GARCH	0.727	0.790	-0.0006	-0.0006	-139.187	-123.185
GO-GARCH	0.703	0.716	-0.0006	-0.0007	-131.318	-103.013
EWMA (94%)	0.666	0.693	-0.0006	-0.0006	--160.538	-107.625
EWMA (94%, 0.1%)	0.105	0.095	-0.0009	-0.0011	-126.436	-95.034
EWMA (94%, 0.2%)	0.040	0.035	-0.0010	-0.0012	-124.905	-96.628
EWMA (92.5%)	0.698	0.728	-0.0006	-0.0006	-161.588	-104.747
EWMA (92.5%, 0.1%)	0.099	0.094	-0.0009	-0.0011	-126.035	-95.281
EWMA (92.5%, 0.2%)	0.036	0.033	-0.0010	-0.0012	-120.426	-97.318
OLS	0.482	0.466	-0.0007	-0.0008	-152.438	-109.638
NAÏVE	0.357	0.330	-0.0008	-0.0009	-140.044	-105.741
50% in-sample period						
	Ederington		**Mean-Variance Utility**		**Certainty Equivalent**	
	FTSE USA/ Sukuk	**Emerging markets/ Sukuk**	**FTSE USA/ Sukuk**	**Emerging markets/ Sukuk**	**FTSE USA/ Sukuk**	**Emerging markets/ Sukuk**
DECO-GARCH	0.427	0.380	-0.002	-0.002	-361.041	-281.512
ADCC-GARCH	0.435	0.346	-0.002	-0.002	-362.192	-280.512
GO-GARCH	0.378	0.330	-0.002	-0.002	-344.091	-307.292
EWMA (94%)	0.394	0.397	-0.002	-0.002	-335.822	-295.004
EWMA (94%, 0.1%)	0.134	0.101	-0.003	-0.003	-278.377	-201.322
EWMA (94%, 0.2%)	0.052	0.037	-0.003	-0.003	-282.031	-202.111
EWMA (92.5%)	0.395	0.402	-0.002	-0.002	-347.781	-311.819
EWMA (92.5%, 0.1%)	0.132	0.102	-0.003	-0.003	-280.008	-203.039
EWMA (92.5%, 0.2%)	0.050	0.036	-0.003	-0.003	-282.042	-202.453
OLS	0.360	0.344	-0.002	-0.002	-315.264	-254.021
NAÏVE	0.252	0.244	-0.003	-0.002	-295.110	-212.379

Notes: The table reports the hedging performance of the hedged portfolio returns. The Ederington value is the difference in variance between hedged and un-hedged portfolios. The mean-variance utility considers the investor's risk aversion value. To account for the investor's resistance to negative skewness and positive excess kurtosis, the certainty equivalent is derived from the return data and an exponential utility function. Higher values are better than lower values.

Table 8.
ρ-values of Minimum Confidence Set - Wavelet Transformed Returns (2-4 days)

	50% in-sample period		75% in-sample period	
	FTSE USA/ Sukuk	**Emerging markets/Sukuk**	**FTSE USA/ Sukuk**	**Emerging markets/Sukuk**
DECO-GARCH	0.002	0.000	0.005	0.000
ADCC-GARCH	0.002	0.000	0.002	0.000
GO-GARCH	0.009	0.000	0.000	0.000
EWMA (94%)	0.002	0.002	0.007	0.000
EWMA (94%, 0.1%)	1.000	1.000	0.007	0.006
EWMA (94%, 0.2%)	0.006	0.007	0.005	0.003
EWMA (92.5%)	0.001	0.001	0.000	0.002
EWMA (92.5%, 0.1%)	0.001	0.005	0.007	1.000
EWMA (92.5%, 0.2%)	0.005	0.003	1.000	0.008
OLS	0.000	0.000	0.000	0.007
NAÏVE	0.000	0.000	0.001	0.005

Notes: This table reports the ρ-values of the minimum confidence set (raw returns). The MCS is used to statistically determine the best-fitting model(s) based on certainty equivalent evaluation. Models that have ρ-values below 0.1 are excluded from the MCS. The shaded areas are the best-fitting models.

4.4. Robustness Check

So far, Sukuk has been used as a diversifier asset (Naeem et al., 2023). Previous studies show that precious metals can hedge the risk of equity. Therefore, the first robustness test is utilizing precious metals as hedging instruments. The data are obtained from *Bloomberg*: Physical Precious Metals Basket Shares ETF (ticker name: GLTR:US), Wahed FTSE USA Shariah ETF (HLAL:US), and iShares MSCI EM Islamic UCITS ETF (ISDE: LN). The daily data cover July 18, 2019 - Oct 25, 2023. All prices are denominated in the US dollars. Tables 10 and 11 show the proposed models are best-equipped for hedging.

The second robustness check is that we used different asset classes. In particular, we apply the spot prices of gold and gold futures (as a hedging instrument for gold). Following previous studies (Jena et al., 2018), the start of the daily data is Jan 11, 2008. The end of the daily data is Oct 25, 2023. Tables 12 outlines that the proposed models are the best-fitting models.

Table 9.
Certainty Equivalent - Wavelet Transformed Returns (64 - 128 days)

	50% in-sample period		75% in-sample period	
	FTSE USA/ Sukuk	Emerging markets/Sukuk	FTSE USA/ Sukuk	Emerging markets/Sukuk
DECO-GARCH	-384.396	-358.580	-205.466	-111.190
ADCC-GARCH	-384.189	-374.019	-203.680	-110.513
GO-GARCH	-386.498	-366.808	-151.467	-143.545
EWMA (94%)	-380.700	-390.245	-156.599	-106.229
EWMA (94%, 0.1%)	-314.494	-248.188	-127.261	-96.676
EWMA (94%, 0.2%)	-314.425	-243.956	-126.070	-98.402
EWMA (92.5%)	-391.904	-411.020	-159.161	-105.112
EWMA (92.5%, 0.1%)	-318.350	-250.542	-126.906	-96.008
EWMA (92.5%, 0.2%)	-315.288	-244.445	-125.639	-98.135
OLS	-371.136	-307.803	-165.662	-106.292
NAÏVE	-329.340	-270.049	-140.483	-108.949

Notes: This table reports the certainty equivalent evaluation which is derived from the return data and an exponential utility function. Higher values are better than lower values. The shaded cells are the best-fitting models.

Table 10.
Certainty Equivalent - Wavelet Transformed Returns (2-4 days)

	50% in-sample period		75% in-sample period	
	FTSE USA/ Metals	Emerging markets/ Metals	FTSE USA/ Metals	Emerging markets/ Metals
DECO-GARCH	-250.214	-240.306	-103.363	-109.451
ADCC-GARCH	-250.214	-241.003	-102.813	-113.084
GO-GARCH	-357.387	-242.378	-109.593	-104.366
EWMA (94%)	-267.092	-240.892	-102.276	-107.782
EWMA (94%, 0.1%)	-281.443	-232.573	-111.388	-100.377
EWMA (94%, 0.2%)	-308.619	-239.428	-89.691	-97.542
EWMA (92.5%)	-262.845	-244.670	-105.782	-113.745
EWMA (92.5%, 0.1%)	-262.090	-223.122	-385.033	-231.257
EWMA (92.5%, 0.2%)	-305.823	-239.934	-87.338	-95.037
OLS	-290.841	-233.018	-104.948	-100.008
NAÏVE	-271.259	-240.285	-155.927	-111.194

Notes: This table reports the certainty equivalent evaluation which is derived from the return data and an exponential utility function. Higher values are better than lower values. The shaded areas are the best-fitting models.

Table 11.
Certainty Equivalent - Wavelet Transformed Returns (64-128 days)

	50% in-sample period		75% in-sample period	
	FTSE USA/ Metals	Emerging markets/ Metals	FTSE USA/ Metals	Emerging markets/ Metals
DECO-GARCH	-321.556	-352.270	-103.921	-127.467
ADCC-GARCH	-321.846	-352.626	-104.709	-127.181
GO-GARCH	-319.225	-375.701	-100.202	-108.124
EWMA (94%)	-335.014	-375.039	-100.029	-107.254
EWMA (94%, 0.1%)	-347.309	-352.049	-110.640	-98.372
EWMA (94%, 0.2%)	-364.461	-331.312	-88.124	-99.423
EWMA (92.5%)	-324.050	-367.135	-101.977	-113.480
EWMA (92.5%, 0.1%)	-316.259	-322.446	-381.371	-228.257
EWMA (92.5%, 0.2%)	-356.621	-320.759	-87.630	-94.859
OLS	-368.722	-378.706	-109.631	-108.990
NAÏVE	-352.227	-458.225	-156.313	-111.815

Notes: This table reports the certainty equivalent evaluation which is derived from the return data and an exponential utility function. Higher values are better than lower values. The shaded areas are the best-fitting models.

Table 12.
Certainty Equivalent - Wavelet Transformed Returns (Gold Spot/Gold Futures)

	2-4 days		64-128 days	
	Spot/Futures 50% in-sample	Spot/Futures 75% in-sample	Spot/Futures 50% in-sample	Spot/ Futures 75% in-sample
DECO-GARCH	-1361	-166	-1504	-770
ADCC-GARCH	-1361	-166	-1512	-770
GO-GARCH	-1361	-166	-1512	-770
EWMA (94%)	-1549	-282	-1374	-312
EWMA (94%, 0.1%)	-1550	-512	-1529	-818
EWMA (94%, 0.2%)	-175	-127	-154	-127
EWMA (92.5%)	-1468	-275	-1306	-308
EWMA (92.5%, 0.1%)	-1731	-482	-1529	-950
EWMA (92.5%, 0.2%)	-173	-130	-154	-130
OLS	-581	-193	-562	-212
NAÏVE	-1762	-342	-1698	-358

Notes: This table reports the certainty equivalent evaluation which is derived from the return data and an exponential utility function. Higher values are better than lower values. The shaded cells are the best-fitting models.

4.5. Analysis

4.5.1 Model Misspecification

This study concludes that simpler models may beat complex models such as GARCH. One of the possible reasons is model misspecification. Following Wang et al. (2015), this research applies simulated data and repeats the analysis of the models based on in-sample data. Monte Carlo simulation generates 10,000 returns. Specifically, this study uses ADCC-GARCH as the true data-generating process

(DGP). Table 13 shows that the true model strategy (ADCC-GARCH) produces the best results. Hence, a model misspecification is one of the reasons why GARCH models do not always provide superior performance in out-of-sample settings.

Table 13.
Certainty equivalent - ADCC-GARCH as DGP

	FTSE USA/Sukuk	Emerging markets/Sukuk
DECO-GARCH	-1504	-352
ADCC-GARCH	-1297	-349
GO-GARCH	-1462	-351
EWMA (94%)	-1561	-358
EWMA (94%, 0.1%)	-1565	-357
EWMA (94%, 0.2%)	-1533	-359
EWMA (92.5%)	-1526	-364
EWMA (92.5%, 0.1%)	-1539	-367
EWMA (92.5%, 0.2%)	-1533	-357
OLS	-1597	-364
NAÏVE	-1473	-353

Notes: This table reports the analysis based on simulated data. Monte Carlo simulation produced 10,000 returns. Specifically, this study used ADCC-GARCH as the true data-generating process (DGP). The table shows that the true model strategy (ADCC-GARCH) produces the best performance.

4.5.2 Contributions to the Literature

This study concerns the methodological gap in the literature. So far, this is the first study that uses EWMA with volatility response to compute a hedge ratio. The second one is related to Islamic hedging. This section presents the analysis concerning those results.

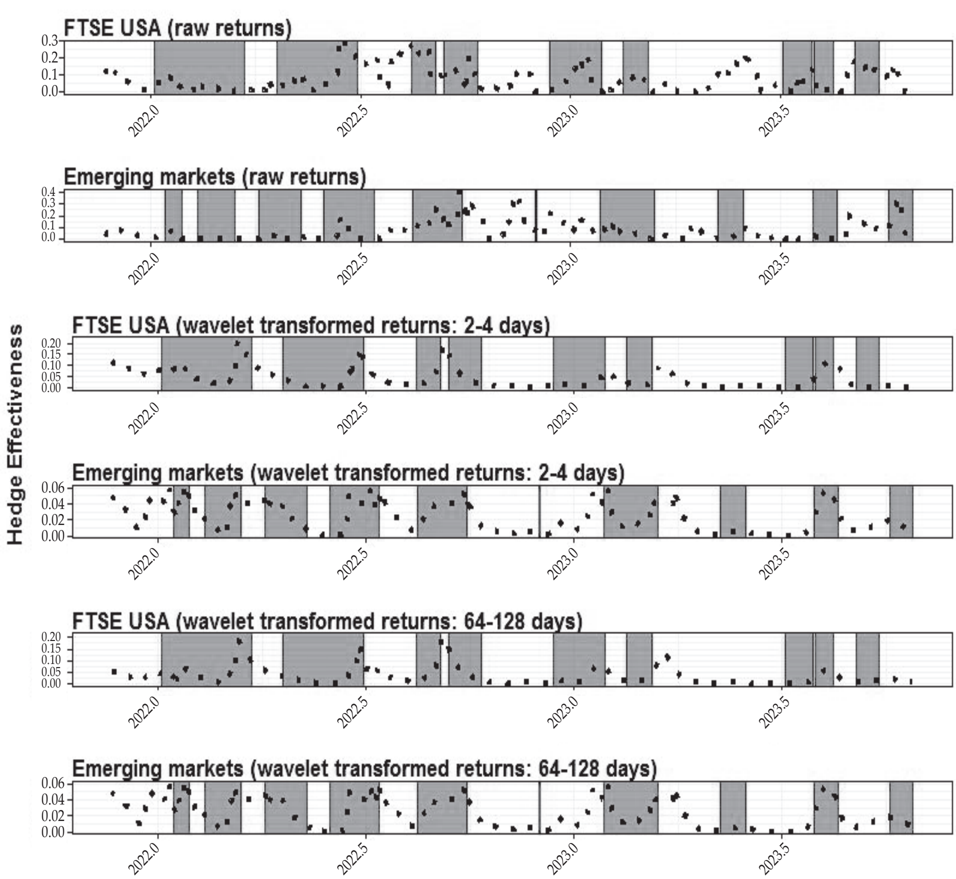
Conventional hedging methods (OLS and naïve approaches) are not always the best method because they do not always predict current market conditions better. Consequently, conventional hedging methods may produce very high kurtosis, implying the models are very risky. Thus, risk managers often prefer to use more risk-sensitive covariance matrix estimates, such as the EWMA and GARCH models.

However, GARCH models have some drawbacks. For example, there may be convergence issues while maximizing the log-likelihood function. Frequently, a long daily return period is required to ensure the model's convergence. If there are insufficient observations, estimates of parameters may lack robustness.

Moreover, the EWMA model gives greater weight to more recent observations. In other words, as extreme returns shift even more into the past as the data window is moved, their significance in the average diminishes. However, the original EWMA model does not consider the *fat*-tailed distribution of the returns. Table 2 shows that the returns of FTSE USA and emerging markets followed a *fat*-tailed distribution. Inspired by Alexander and Dakos (2023), the proposed models modify the volatilities produced by the original EWMA to account for *fat*-tailed distribution. The proposed models (EWMA (94%, 0.1%), EWMA (94%, 0.2%), EWMA (92.5%, 0.1%), and EWMA (92.5%, 0.2%) have significantly improved the kurtosis of the original EWMA if we take into account the heterogeneous timing preferences.

The results suggest that to minimize the risk of Islamic equity, traders or hedgers need to short Sukuk or green bonds. However, conventional short selling is not permitted from the Islamic perspective. One possible scheme for Islamic short-selling is W'ad-based structures (Calder, 2010). Bursa Malaysia has initiated this approach.

This research contributes to the existing literature in the following ways: First, this study shows the outperformance of the EWMA model adjusted for a volatility response over a widely used method in the literature. Second, this study resolves contradictory outcomes regarding the EWMA models. In other words, the EWMA method produces better forecasts of hedge ratios provided that the models incorporate volatility responses. Hence, this study extends the previous studies (Liu et al., 2020; Nekhili & Sultan, 2022; Silahli et al., 2021). The outperformance of a less complex model is also apparent in Sharma & Karmakar (2023).



Notes: Red shades show bear regimes. The strategy is EWMA (0.925, 0.002). The hedge effectiveness is based on 50% in-sample data. 1 indicates perfect hedge.

Figure 4.
Dynamic Hedge Effectiveness

4.5.3 Dynamic Hedge Effectiveness

The dynamic hedge effectiveness is another tool to measure the economic significance of hedging. This study follows a research by Li & Zakamulin (2020) to determine the dates of the bear markets. Figure 4 illustrates that Sukuk could minimize the risk of Islamic equity during the bear markets (red shaded areas). To save space, we only provide the dynamic hedge based on EWMA (0.925, 0.002) strategy. Interestingly, Sukuk could reduce the risk of Islamic equity in the USA more than the risk of Islamic equity in the emerging markets. A possible explanation is that the news impact curve of Islamic equity in the USA is an extremely asymmetric graph (see Figure 1).

4.5.4. Sukuk is a Diversifier Asset

Additionally, this study investigates Sukuk's capacity for diversification in the context of severely negative fluctuations in Islamic equities markets. We use the following regression (Bouri, Lucey, & Roubaud, 2020; Drobetz, Schröder, & Tegtmeier, 2020):

$$DCC_t = m_0 + m_1 D\left(r_{Islamic\ equity}\ q_{10}\right) + m_2 D\left(r_{Islamic\ equity}\ q_5\right) + m_3 D\left(r_{Islamic\ equity}\ q_1\right) + v_t \tag{19}$$

DCC_t is the dynamic pairwise correlation between Sukuk and Islamic equity. $D(r_{Islamic\ fund}\ q_{10})$, $D(r_{Islamic\ equity}\ q_5)$, and $(r_{Islamic\ equity}\ q_1)$ are dummy variables that indicate extreme shocks in the lower 10th percentile, 5th percentile, and 1st percentile, respectively. Sukuk is a diversifier for Islamic equity if m_0 is positive and significant. If m_0 is negative (zero), Sukuk is strong (weak) hedge. Sukuk is a safe-haven asset for Islamic equity if m_0 is negative and the m_1, m_2, and m_3 are also negative or zero.

Table 14 provides evidence that Sukuk shows similar diversification abilities, regardless of the method used. Particularly, Sukuk is a strong diversifier for the USA and emerging equity markets because the coefficients on the constant parameters m_0 are significantly positive. It is worth noting that EWMA (94%, 0.1%) and EWMA (94%, 0.2%) have the same results with EWMA (94%) because the parameter of volatility response (η) only affects variance and covariance. In addition, naïve method is not included since the method does not have time-varying pairwise correlation. Further, the results from wavelet-transformed returns are similar to the results of Table 14.

Table 14.
Diversification Abilities of Sukuk

75 % in-sample period				
	Hedge (m_0)	**10% quintile (m_1)**	**5% quintile (m_2)**	**1% quintile (m_3)**
DECO-GARCH				
FTSE USA/Sukuk	0.155***	0.074**	0.020	0.066
Emerging markets/Sukuk	0.210***	0.013	0.001	0.076
ADCC-GARCH				
FTSE USA/Sukuk	0.155***	0.074	0.020	0.069
Emerging markets/Sukuk	0.208***	0.008	0.004	0.026
GO-GARCH				
FTSE USA/Sukuk	0.280***	0.052**	0.005	0.008
Emerging markets/Sukuk	0.159***	-0.047	0.101	-0.070
EWMA with λ = 94%				
FTSE USA/Sukuk	0.087***	0.093	-0.055	0.127
Emerging markets/Sukuk	0.144***	-0.033	0.113	0.117
EWMA with λ = 92.5%				
FTSE USA/Sukuk	0.084***	0.092	-0.083	0.154
Emerging markets/Sukuk	0.142***	-0.044	0.130	0.255
Conventional method				
FTSE USA/Sukuk	0.277***	0.010	0.036	0.050
Emerging markets/Sukuk	0.179***	0.006	0.001	0.018
50 % in-sample period				
	Hedge (m_0)	**10% quintile (m_1)**	**5% quintile (m_2)**	**1% quintile (m_3)**
DECO-GARCH				
FTSE USA/Sukuk	0.242***	0.033**	0.013	0.028
Emerging markets/Sukuk	0.206***	0.001	0.004	-0.001
ADCC-GARCH				
FTSE USA/Sukuk	0.245***	0.030	0.012	0.028
Emerging markets/Sukuk	0.229***	-0.011	-0.006	-0.008
GO-GARCH				
FTSE USA/Sukuk	0.275***	0.025	-0.026	0.124**
Emerging markets/Sukuk	0.229***	-0.011	-0.006	-0.008
EWMA with λ = 94%				
FTSE USA/Sukuk	0.162***	0.045	-0.025	0.187**
Emerging markets/Sukuk	0.184***	-0.017	0.050	0.047
EWMA with λ = 92.5%				
FTSE USA/Sukuk	0.159***	0.049	-0.030	0.108**
Emerging markets/Sukuk	0.180***	-0.021	0.049	0.066
Conventional method				
FTSE USA/Sukuk	0.268***	0.038**	0.018	0.009
Emerging markets/Sukuk	0.169***	-0.009	-0.010	0.008

Notes: This table shows the coefficients from the following regression: $DCC_t = m_0 + m_1 D(r_{Islamic\,equity}\, q_{10}) + m_2 D(r_{Islamic\,equity}\, q_5) + m_3 D(r_{Islamic\,equity}\, q_1) + v_t$. Sukuk is a diversifier for Islamic equity if m_0 is positive and significant. If m_0 is negative (zero), Sukuk is strong (weak) hedge. Sukuk is a safe-haven asset for Islamic equity if m_0 is negative and the m_1, m_2, and m_3 are also negative or zero. ***, **, and * show statistical significance at 0.01, 0.05, and 0.10 levels, respectively.

V. CONCLUSION AND RECOMMENDATION

5.1. Conclusion

In this paper, we propose the EWMA-based models to compute hedge ratios. Further, two forms of data splitting and a rolling-window analysis are carried out to reduce data mining bias. This work also implements the widely-used methods such as DECO-GARCH, GO-GARCH, asymmetric DCC-GARCH, naïve approach, and linear regression.

The first part of this research evaluates the hedging performance based on raw returns. The minimum confidence set outlines that no single hedging model dominates other strategies based on certainty equivalent evaluation. The second part of this article mainly discusses the performance of the wavelet-based hedge ratios. The proposed models not only improve the original EWMA but also outperform the others.

The limitation of this research is that there is no definitive way to select the best-fit λ and η in our proposed models. Another shortcoming is that sharia compliant short selling is required to minimize the risk of Islamic equity. Unfortunately, the markets for Islamic instruments are still relatively limited.

5.2. Recommendation

This study generates essential suggestions for different stakeholders. First, this research encourages investors to apply a more straightforward model as long as the model considers the existence of a *fat*-tailed return distribution. The proposed approach is an EWMA (λ, η) model, where η is the volatility response, and λ is the smoothing constant. Second, the results of this work also encourage policymakers to support Islamic hedging further using Islamic exchange-traded funds. Sharia rules do not apply to the tradability of an option. On the other hand, if the option's incentive component is taken into account, a different, Sharia-compliant incentive may be provided in place of the option. It is possible to design a unilateral W'ad (promise/undertaking) consent to provide incentive elements similar to an option. Alternatively, the option seller may agree to exchange equity with the buyer for a set period of time at a predetermined price or market rate. The AAOIFI standard No. 49, which specifies the following, *"It is permissible for a party to promise to enter into a commutative contract in the future"*, supports hedging in Islamic finance (State Bank of Pakistan, 2020).

For future research, it is recommended to use penalized quintile regression on EWMA (λ, η) models since there is no definitive way to select the best-fit λ and η. We also suggest to test the models on non-sharia assets.

REFERENCES

Ahmad, W., Sadorsky, P., & Sharma, A. (2018). Optimal hedge ratios for clean energy equities. *Economic Modelling*, *72*, 278–295. https://doi.org/10.1016/j.econmod.2018.02.008

Alam, M., & Ansari, V. A. (2020). Are Islamic indices a viable investment avenue? An empirical study of Islamic and conventional indices in India. *International Journal of Islamic and Middle Eastern Finance and Management*, *13*(3), 503–518.

Alexander, C., & Barbosa, A. (2008). Hedging index exchange traded funds. *Journal of Banking and Finance*, *32*(2), 326–337.

Alexander, C., & Dakos, M. (2023). Assessing the accuracy of exponentially weighted moving average models for Value-at-Risk and Expected Shortfall of crypto portfolios. *Quantitative Finance*, *23*(3), 393–427.

Antonakakis, N., Cunado, J., Filis, G., Gabauer, D., & de Gracia, F. P. (2020). Oil and asset classes implied volatilities: Investment strategies and hedging effectiveness. *Energy Economics*, *91*, 104762. https://doi.org/10.1016/j.eneco.2020.104762

Antonakakis, N., Cunado, J., Filis, G., Gabauer, D., & Perez de Gracia, F. (2018). Oil volatility, oil and gas firms and portfolio diversification. *Energy Economics*, *70*, 499–515. https://doi.org/10.1016/j.eneco.2018.01.023

Arif, M., Naeem, M. A., Hasan, M., M Alawi, S., & Taghizadeh-Hesary, F. (2022). Pandemic crisis versus global financial crisis: Are Islamic stocks a safe-haven for G7 markets? *Economic Research-Ekonomska Istraživanja*, *35*(1), 1707–1733.

Bandhu Majumder, S. (2022). Searching for hedging and safe haven assets for Indian equity market – a comparison between gold, cryptocurrency and commodities. *Indian Growth and Development Review*, *15*(1), 60–84.

Batten, J. A., Kinateder, H., Szilagyi, P. G., & Wagner, N. F. (2019). Hedging stocks with oil. *Energy Economics, 93*, 104422.

Baur, D. G., & Lucey, B. M. (2010). Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold. *The Financial Review*, *45*(2010), 217–229.

Bauwens, L., Laurent, S., & Rombouts, J. V. K. (2006). Multivariate GARCH models: A survey. *Journal of Applied Econometrics*, *21*(1), 79–109.

Bouri, E., Lucey, B., & Roubaud, D. (2020). Cryptocurrencies and the downside risk in equity investments. *Finance Research Letters*, *33*, 101211. https://doi.org/10.1016/j.frl.2019.06.009

Bouri, E., Shahzad, S. J. H., Roubaud, D., Kristoufek, L., & Lucey, B. (2020). Bitcoin, gold, and commodities as safe havens for stocks: New insight through wavelet analysis. *The Quarterly Review of Economics and Finance*, *77*, 156–164. https://doi.org/10.1016/j.qref.2020.03.004

Brixton, A., Brooks, J., Hecht, P., Ilmanen, A., Maloney, T., & McQuinn, N. (2023). A changing stock–bond correlation: Drivers and implications. *The Journal of Portfolio Management, 49*(4), 64-80.

Calder, R. (2010). Short-selling repilication in Islamic finance: Innovation and debate in Malaysia and beyond. In *Current Issues in Islamic Banking and Finance* (pp. 277–315). https://doi.org/10.1142/9789812833938_0014

Cappiello, L., Engle, R., & Sheppard, K. (2006). Asymmetric dynamics in the correlations of global equity and bond returns. *Journal of Financial Econometrics*, *4*(4), 537–572.

Catania, L., & Grassi, S. (2022). Forecasting cryptocurrency volatility. *International Journal of Forecasting*, *38*(3), 878–894.

Dharani, M., Hassan, M. K., Rabbani, M. R., & Huq, T. (2022). Does the Covid-19 pandemic affect faith-based investments? Evidence from global sectoral indices. *Research in International Business and Finance*, *59*, 101537. https://doi.org/10.1016/j.ribaf.2021.101537

Drobetz, W., Schröder, H., & Tegtmeier, L. (2020). The role of catastrophe bonds in an international multi-asset portfolio: Diversifier, hedge, or safe haven? *Finance Research Letters*, *33*, 101198. https://doi.org/10.1016/j.frl.2019.05.016

Ederington, L. H. (1979). The hedging performance of the new futures markets. *The Journal of Finance*, *34*(1), 157–170.

Engle, R., & Kelly, B. (2012). Dynamic equicorrelation. *Journal of Business & Economic Statistics*, *30*(2), 212–228.

Ghaemi Asl, M., & Rashidi, M. M. (2021). Dynamic diversification benefits of Sukuk and conventional bonds for the financial performance of MENA region companies: empirical evidence from COVID-19 pandemic period. *Journal of Islamic Accounting and Business Research*, *12*(7), 979–999.

Hamma, W., Ghorbel, A., & Jarboui, A. (2021). Hedging Islamic and conventional stock markets with other financial assets: Comparison between competing DCC models on hedging effectiveness. *Journal of Asset Management*, *22*(3), 179–199.

Hansen, P. R., Lunde, A., & Nason, J. M. (2011). The model confidence set. *Econometrica*, *79*(2), 453–497.

Hasan, Z. (2018). Academic sociology: The alarming rise in predatory publishing and its consequences for Islamic economics and finance. *ISRA International Journal of Islamic Finance*, *10*(1), 6–18.

Injadat, E. (2018). the Practical Model of Hedging in Islamic Financial Markets. *International Journal of Economics, Commerce and Management United Kingdom*, *6*(6), 134–140.

Ismal, R. (2022). Assessing the application of Islamic and conventional hedgings in Indonesia. *International Journal of Islamic and Middle Eastern Finance and Management*, *15*(1), 32–47.

Izadi, S., & Hassan, M. K. (2018). Portfolio and hedging effectiveness of financial assets of the G7 countries. *Eurasian Economic Review*, *8*(2), 183–213.

Jalkh, N., Bouri, E., Vo, X. V., & Dutta, A. (2020). Hedging the risk of travel and leisure stocks: The role of crude oil. *Tourism Economics, 27*(7), 1337-1356.

Jena, S. K., Tiwari, A. K., & Roubaud, D. (2018). Comovements of gold futures markets and the spot market: A wavelet analysis. *Finance Research Letters*, *24*, 19–24. https://doi.org/https://doi.org/10.1016/j.frl.2017.05.006

Jeribi, A., & Fakhfekh, M. (2021). Portfolio management and dependence structure between cryptocurrencies and traditional assets: evidence from FIEGARCH-EVT-Copula. *Journal of Asset Management*, *22*(3), 224–239.

Kamara, A., Korajczyk, R. A., Lou, X., & Sadka, R. (2016). Horizon Pricing. *The Journal of Financial and Quantitative Analysis*, *51*(6), 1769–1793.

Karim, S., & Naeem, M. A. (2022). Do global factors drive the interconnectedness among green, Islamic and conventional financial markets? *International Journal of Managerial Finance*, *18*(4), 639–660.

Kroner, K. F., & Sultan, J. (1993). Time-varying distributions and dynamic hedging with foreign currency futures. *The Journal of Financial and Quantitative Analysis*, *28*(4), 535–551.

Li, X., & Zakamulin, V. (2020). Stock volatility predictability in bull and bear markets. *Quantitative Finance*, *20*(7), 1149–1167.

Liu, W., Semeyutin, A., Lau, C. K. M., & Gozgor, G. (2020). Forecasting value-at-risk of cryptocurrencies with riskmetrics type models. *Research in International Business and Finance*, *54*. https://doi.org/10.1016/j.ribaf.2020.101259

Naeem, M. A., Mbarki, I., Alharthi, M., Omri, A., & Shahzad, S. J. H. (2021). Did COVID-19 impact the connectedness between green bonds and other financial markets? evidence from time-frequency domain with portfolio implications. *Frontiers in Environmental Science*, *9*(May), 1–15. https://doi.org/10.3389/fenvs.2021.657533

Naeem, M. A., Raza Rabbani, M., Karim, S., & Billah, S. M. (2023). Religion vs ethics: Hedge and safe haven properties of Sukuk and green bonds for stock markets pre- and during COVID-19. *International Journal of Islamic and Middle Eastern Finance and Management*, *16*(2), 234–252.

Narayan, P. K., & Sharma, S. S. (2016). Intraday return predictability, portfolio maximisation, and hedging. *Emerging Markets Review*, *28*, 105–116. https://doi.org/10.1016/j.ememar.2016.08.017

Nekhili, R., & Sultan, J. (2022). Hedging Bitcoin with conventional assets. *Borsa Istanbul Review*, *22*(4), 641–652. https://doi.org/10.1016/j.bir.2021.09.003

Nguyen, T. T. H., Naeem, M. A., Balli, F., Balli, H. O., & Vo, X. V. (2021). Time-frequency comovement among green bonds, stocks, commodities, clean energy, and conventional bonds. *Finance Research Letters*, *40*, 101739. https://doi.org/10.1016/j.frl.2020.101739

Qureshi, S., Aftab, M., Bouri, E., & Saeed, T. (2020). Dynamic interdependence of cryptocurrency markets: An analysis across time and frequency. *Physica A: Statistical Mechanics and Its Applications*, *559*, 125077. https://doi.org/10.1016/j.physa.2020.125077

Ryan, L. (2021). Bonds don't need to be negatively correlated with equities. *The Journal of Investing, 30*(60), 70-80.

Samitas, A., Papathanasiou, S., & Koutsokostas, D. (2021). The connectedness between Sukuk and conventional bond markets and the implications for investors. *International Journal of Islamic and Middle Eastern Finance and Management*. https://doi.org/10.1108/IMEFM-04-2020-0161

Sharma, U., & Karmakar, M. (2023). Measuring minimum variance hedging effectiveness: Traditional vs. sophisticated models. *International Review of Financial Analysis*, *87*, 102621. https://doi.org/10.1016/j.irfa.2023.102621

Silahli, B., Dingec, K. D., Cifter, A., & Aydin, N. (2021). Portfolio value-at-risk with two-sided Weibull distribution: Evidence from cryptocurrency markets. *Finance Research Letters*, *38,* 101425.

Sisodia, G., Joseph, A., & Dominic, J. (2022). Whether corporate green bonds act as armour during crises? Evidence from a natural experiment. *International Journal of Managerial Finance*, *18*(4), 701–724.

Stamos, M. (2022). Forecasting stock market volatility. *The Journal of Portfolio Management*, *49*(3), 129-137.

State Bank of Pakistan. (2020). Adoption of AAOIFI Shariah Standard No. 49. Retrieved October 25, 2023, from https://www.sbp.org.pk/ibd/2020/C3-Annex-A.pdf

Van der Weide, R. (2002). GO-GARCH: A Multivariate generalized orthogonal GARCH model. *Journal of Applied Econometrics*, *17*(5), 549–564.

Wang, Y., Wu, C., & Yang, L. (2015). Hedging with futures: Does anything beat the naïve hedging strategy? *Management Science*, *61*(12), 2870–2889.

Yousaf, I., Ali, S., Naveed, M., & Adeel, I. (2021). Risk and return transmissions from crude oil to Latin American stock markets during the crisis: Portfolio implications. *SAGE Open*, *11*(2). https://doi.org/10.1177/21582440211013800

Zghal, R., Melki, A., & Ghorbel, A. (2022). Do commodities hedge regional stock markets at the same effectiveness level? Evidence from MGARCH models. *International Journal of Emerging Markets*, (ahead-of-print).

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