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A MARKOV CHAIN MODEL FOR ISLAMIC MICRO-FINANCING

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ABSTRACT

This paper introduces a Markov chain model for Islamic micro-financing, especially *mudarabah* and *murababah* contracts. *Mudarabah* and *murabahah* are two Islamic micro-financing contracts that have enormous potential in creating a balance between the monetary and sharia sector because the two products are moving to manage the business sector, which undoubtedly adds value to the economic movement directly. On the other hand, both contracts have the potential to cause problems in their implementation, notably the asymmetric information that consists of adverse selection and moral hazard. We propose the Markov chain model as a solution for the Islamic banks to reduce the risk due to adverse selection and moral hazard in *mudarabah* and *murabahah* contracts. In our model, we also propose a mechanism to avoid strategic default in a *mudarabah* contract. We observed two different probabilities of an applicant to become a beneficiary to find the solution to the problems. The results of this study reveal that the bank could decrease the probability of an applicant to become a beneficiary to reduce the adverse selection and moral hazard in *mudarabah* and *murabahah* contracts.

Keywords: Markov Chain, Mudarabah, Murabahah, Adverse Selection, Moral Hazard.
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I. INTRODUCTION

1.1. Background

Microcredit has developed in poor countries as well as in richer countries, including those of Muslim-majority. High unemployment, poverty, and difficulties of financial access have an impact on increasing demand for microcredit in the Muslim-majority countries, most notably Bangladesh and Indonesia (Khaled, 2011). Along with the development of microcredit in Muslim-majority countries, controversies over the conventional microcredit systems are inevitable. Islam forbids *riba* (usury/interest). Qur'an explicitly explains this. Many Islamic (micro) finance institutions are developing Islamic micro-financing now. They develop Islamic finance in several types of contracts according to the client's requirements. Some type of contracts in Islamic micro-financing is *murabahah*, *ijarah*, *musyarakah*, and *mudarabah.*

Mudarabah and *murabahah* are two Islamic micro-financing contracts that have enormous potential in creating a balance between the monetary and sharia sectors because both products have gradually infiltrated the business sector, which undoubtedly adds value to the economic movement directly. As the need for Islamic micro-financing increases, various problems arise in its implementation, including in *mudarabah* and *murabahah* contracts. The most common problem is asymmetric information, which consists of adverse selection and moral hazard. In a paper (Redone and Yaman, 2018), they have done a study in the Indonesian Islamic banking industry about asymmetric information problems in *mudarabah* and *murabahah* contracts. Previous studies have discussed such problems (Tag El-Din, 2008; Ismal, 2009; Kamarudin & Ismal, 2013; Yousfi, 2013; Nouman & Ullah, 2014; Fakir & Thiout, 2016; Sapuan, 2016; and Jouaber & Mehri, 2017). In some of the papers, it was explained that adverse selection occurs because the banks do not know the applicant's character so that the banks make mistakes in choosing clients, while moral hazard occurs when the clients do not give the right financial report. Such problems are risky for Islamic banks. Adverse selection occurs before contract transactions between bank and applicant, while moral hazard occurs during the contract period possibly due to adverse selection.

In this paper, we propose the Markov chain model for *mudarabah* and *murabahah* contracts. A definition of Markov chain by Mills in Chan and Lenard (2012): *"Markov chains are a mathematical model that use concepts from probability to describe how a system changes from one state to another"*. By applying this model in a *mudarabah* contract, the Islamic bank can estimate or determine the number of applicants to become beneficiary to reduce adverse selection or moral hazard. Furthermore, in *mudarabah* and *murabahah* contracts, the Islamic bank can estimate the total profit of beneficiary and the total profit of the bank, calculating the proportion of beneficiaries in a population at equilibrium. By recognizing the total profit of beneficiary and bank, and the proportion of beneficiaries at an equilibrium condition, the Islamic bank can decide to reduce or avoid adverse selection and moral hazard.

1.2. Objective

As far as the objective of this paper is concerned, we propose the Markov chain model as a solution for the bank to reduce the risk due to adverse selection and moral hazard in *mudarabah* and *murabahah contracts*. In this model, the bank can calculate the expectation of beneficiary profits and the expectation of bank profits during some contract periods, both affected by certain probabilities. We also propose a mechanism to avoid a moral hazard in a *mudarabah* contract. The Markov chain model also allows a system of sustainable credit for the beneficiary who is successful in reimbursing the credit, as expected by beneficiaries.

This paper is organized as follows. Section II briefly reviews about the mathematical model of *mudarabah* and *murabah* contracts. Section III discusses Markov chain model in *mudarabah* and *murabahah* contracts. Section IV, we calculate the proportion of the beneficiaries in a population at equilibrium. Section V, we propose the condition of the absence of strategic default in a *mudarabah* contract. In section VI, we show the simulation results of models and analyze the results. Finally, section VII concluded the study and give the recommendations for future study.

II. LITERATURE REVIEW

2.1. Background Theory

2.1.1. Markov Chain

Let $\{X_0, X_1, X_2,\ldots\}$ be a sequence of discrete random variables. Then $\{X_0, X_1, X_2,\ldots\}$ is Markov chain if

$$\mathbb{P}(X_{t+1} = j | X_t = i, X_{t-1} = i_{t-1}, \ldots, X_1 = i_1, X_0 = i_0) = \mathbb{P}(X_{t+1} = j | X_t = i) = \mathbb{P}_{ij} \qquad (1)$$

for all t=1,2,3,… and all states $i_0, i_1, \ldots, i_{t-1}, i, j$.

We can interpret equation (1) that, the conditional distribution of any future state X_{t+1}, given the past states $X_0, X_1, \ldots, X_{t-1}$ and the present X_t, is independent of the past states and depends only on the present state, which is called the Markov property. The value P_{ij} represents the probability that the process will, when in state i, next make a transition into state j.[1]

As far as the application of Markov chain is concerned particularly in a contract of Islamic micro-financing, the model is built by determining the probabilities of applicant's or beneficiary's movements, such as the probability of an applicant to become a beneficiary, the probability of a client succeeding in reimbursing a credit, and the probability of a client fails to reimburse a credit. The probability of an applicant to become a beneficiary will affect the probability of adverse selection, while the probability of the client fails to reimburse a credit is the probability of risk, it can occur due to adverse selection or moral hazard. The bank can determine the probability of an applicant to become a beneficiary from applications of loans.

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1 The references of Markov chain are from Ross (1996) and Fewster (2014).

2.1.2. Mudarabah

A *Mudarabah* contract is a form of partnership between *rabb al-mal* (capital provider) who contributes capital and *mudarib* (beneficiary) who contributes efforts in the form of managerial skill or work. The profit of business is shared between the capital provided and the beneficiary according to the mutually agreed profit sharing ratio in the contract. If the beneficiary loses then losses are born solely by the capital provider, provided such loss is not due to the beneficiary's negligence (Bank Negara Malaysia, 2009).

Example 1:
A beneficiary makes a *mudarabah* contract with an Islamic bank that agrees to provide RM100000 financing. Profit sharing between the beneficiary and the bank in a ratio of 70:30 respectively. If the profit is RM40000, the beneficiary recovers RM100000 capital and shares RM12000 profit. If the loss is RM20000, the bank bears the loss of RM20000, and the beneficiary covers the RM180000 capital.

2.1.3. Murabahah

Murabahah is the most popular Islamic finance contract used in business and asset financing. The bank purchases the goods to the beneficiary, then the beneficiary will pay according to the price and date agreed by two parties (Hussain et al, 2015).

Example 2:
The bank buys \$100000 worth of wood for a customer who is a manufacturer. According to the agreement of two parties, the manufacturer must pay to the bank \$120000 for one year (Jamaldeen, undated).

2.2. Previous Studies

Khodr et al (2009) introduced a Markov Chain model for conventional microcredit in a paper about a mathematical model for microlending. In this paper, they calculated the expected total profit of the borrower. They also presented the condition of absence of strategic default to avoid failure to reimburse a loan, introduced by Tedeschi (2006).

On the other hand, some studies proposed asymmetric information problem solving by using the mathematical modeling in *mudarabah* and *mudarabah* contract.

Jouaber and Mehri (2017) calculated the expected payoff to Islamic Venture Capital (IVC) and the beneficiary. They calculated it in two cases, without adverse selection, and under adverse selection. They optimized profit sharing ratio to avoid the adverse selection problem between the IVC and the beneficiary, and to improve the profit of a venture.

Studies done by Yousfi (2013), he presented the model without moral hazard and the model under moral hazard in a *mudarabah* contract. He optimized the contract under moral hazard by maximizing the expected gain of the beneficiary.

Ismal (2009) presented a model of *mudarabah* contract by making the selection criteria model when the Islamic banking selects the business partners to become a client and build the model to make a financial decision in *mudarabah* contract. As far as the main purpose of the research is concerned, he wanted to find models for minimizing moral hazard because some clients wanted to get profit by pretending to be a default. The Islamic banks would investigate the clients about moral hazard. He found formulas to calculate the profit when the moral hazard is not found and the moral hazard is found by the bank's investigations.

Other than that, other papers discuss the mathematical model of *mudarabah* contract and *murabahah* contracts.

The model presented by Tag El-Din (2008) is the comparison of return and risk model of *mudarabah* contract with four other contracts of the return and risk models, namely the pure borrowing model, the partial borrowing contract, the pure hiring contract, and the partial hiring contract. For determining the risk and return models of types of contracts, he used the expected (mean) formula for calculating the return and the variance formula for calculating the risk. In this research, *mudarabah* is used as a comparison for the other four types of contracts. One of the results of research is the profit and sharing of *mudarabah* cannot be manipulated freely through monetary policy without affecting the client's interest in choosing *mudarabah* contract.

Moreover, Zandi & Arifin (2012) described the model of the practical framework of *mudarabah* in Iran. The Iranian banks offer installment sales facility for buying commodity. They showed the profit of bank formula and the monthly installments amount formula used by the Iranian banks.

On the other hand, Kamarudin and Ismail (2013) showed two models of *mudarabah* contracts. The first model describes the Islamic bank as a capital provider, and the second model the depositor deposit money at Islamic bank for a *mudarabah* contract. Therefore, the bank can observe the outcomes, and the customer only knows about the profit sharing ratio. They used the probability distribution on actual profit/income to build the models. The result of the research, the Islamic bank could get maximum profit and provide an incentive to be an entrepreneur if the Islamic bank receives higher profit than their expectation from the project.

Omar and Jaffar (2016) proposed a continuous model stochastic *mudarabah* investment by creating the stochastic differential equation to find the solution of equity of the capital provider and the equity of the entrepreneur. By using the model, the investment of parties will be predicted when the capital is invested in the stock market.

Further, Fakir and Thiuoat (2016) applied the Hersanyi model to create a model of *mudarabah* contract. They used probability theory to build the model. They calculated the sharing ratio of *mudarabah* for Islamic banks with high expertise and the Islamic bank with low expertise, and then they calculated the expected profit of the client.

Jatmiko et al. (2017) built three models in *mudarabah* contract, namely the modeling the supply side, the modelling the consumers, and the market clearing condition. In the modeling, the supply side and the modelling the consumers maximized the present value of its future cash flow and found the solution conservative, the internal optimum discount rate, and the internal optimum discount rate for consumers. They also found the solution of the third model, i.e., market cleaning condition to evaluate the optimal *mudarabah* discount rate.

III. METHODOLOGY

3.1. Data

In simulating our model, we used data of example 1 for simulations of the Markov chain model of *mudarabah* contracts and we used data of example 2 for the simulations of the Markov chain model of *murabahah* contracts. Additionally, we chose the value of variables that matched the needs of our model. We simulated the model by using SCILAB, which provides a tool (as a command) to call the Markov chain to generate data of the dynamic evolution of the states.

3.2. Model Development

3.2.1. Markov Chain Model of A Mudarabah Contract

In this section, we will build a model of *mudarabah* contract by using the Markov chain. In this model, there are two states: A as the state of being an applicant, and B a beneficiary. The Islamic bank offers a loan to an applicant who becomes a beneficiary with probability γ, she [2] passes state A to state B. If she is unsuccessful to become a beneficiary, she stays at state A with probability 1-γ, she can apply again to ask for a loan. A beneficiary can repay her loan, she stays at state B with probability 1-ε, and she can get a loan again. If she loses, she returns to state A to ask for a loan again with probability ε. Such rules can be summarized in a Markov chain $(X_t)_{t\in N}$ with the set of states $S{:=}\{A,B\}$, where $X_t \in S$. The Markov chain model of *mudarabah* contract can be illustrated by the following picture:

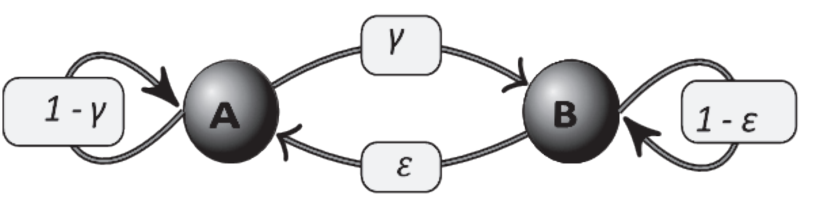

Figure 1.
Markov Chain Model of *Mudarabah* Contract

We can also describe the model into the transition matrix P_d:

$$P_d = \begin{pmatrix} 1-\gamma & \gamma \\ \varepsilon & 1-\varepsilon \end{pmatrix}.$$

We consider the case where the bank can give a loan K for a beneficiary. The beneficiary will do some production activities with the loan. To illustrate, in the production activities, she spends some money for consumption c. After completing the production activities, she gets income $Y(K)$.

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[2] Muhammad Yunus is the pioneer of microcredit. He founded Grameen Bank that lent money to the poorest especially the women in Bangladesh in 1976. In this paper, we change the word “beneficiary” with the word “she”.

In the case where the beneficiary gets a profit from her business $Y(K)$-(c+K), she must divide the profit with the bank in a certain value depending on the contract. We consider a bank's portion for profits as ρ and a beneficiary's portion of profits as 1-ρ, so she must reimburse the capital K plus $\rho(Y(K)-(c+K))$. The beneficiary gains profit, $(1-\rho)(Y(K)-(c+K))$. In the case where the beneficiary loses, where $c+K>Y(K)$, she gains nothing and the bank loses $(c+K)-Y(K)$.

In the *mudarabah* contract, we consider the financial flow function f:SxS→R. The beneficiary profits for one period in *mudarabah* contract is:

$$f_l(X_{t-1}, X_t) = \begin{cases} (1-\rho)(Y(K)-(c+K)), & \&if\ (B,B) \\ 0, & \&if\ not. \end{cases}$$

Supposing δ the discount for the time of one loan, then the expected total profit of beneficiary at the state x and the time t in *mudarabah* contract is:

$$W_l(x) = \sum_{t\geq 1} \delta^t f_l(X_{t-1}, X_t).$$

The bank profits for one period in *mudarabah* contract is:

$$f_k(X_{t-1}, X_t) = \begin{cases} \rho(Y(K)-(c+K)), & \&if\ (B,B) \\ Y(K)-(c+K), & \&if\ (B,A) \\ 0, & \&if\ not. \end{cases}$$

The expected total profit of bank at the state x and the time t in *mudarabah* contract is:

$$W_k(x) = \sum_{t\geq 1} \delta^t f_k(X_{t-1}, X_t).$$

The expected profit of beneficiary for one period in *mudarabah* contract is:

$$E\left[f_l(X_{t-1}, X_t)|_{X_{t-1}=B}\right] = (1-\varepsilon)(1-\rho)(Y(K)-(c+K))$$

In addition, the expected profit of bank for one period in *mudarabah* contract is:

$$\begin{aligned} E\left[f_k(X_{t-1}, X_t)|_{X_{t-1}=B}\right] &= (1-\varepsilon)\rho\big(Y(K)-(c+K)\big) + \varepsilon\big(Y(K)-(c+K)\big) \\ &= ((1-\varepsilon)\rho + \varepsilon\big(Y(K)-(c+K)\big). \end{aligned}$$

If the expected profit of beneficiary for one period is equal to the expected profit of bank for one period, then we can calculate ε (the probability of the risk), if we fix ρ (the bank's portion of profits):

$$\varepsilon = \frac{2\rho-1}{2\rho-2}. \tag{2}$$

3.2.2. Markov Chain Model of Murabahah Contract

In the model of *murabahah* contract, a beneficiary can ask one-unit goods over a period of time t. Three states are involved: A as the state of being an applicant, B a beneficiary, and D a debtor. The bank gives property to an applicant who becomes a beneficiary with probability γ, she then moves from state A to state B. If not, she stays at state A with probability 1-γ, she can apply again to get a property. A beneficiary can repay her property, in which she can stay at state B with probability β, and she can receive the other property. If she cannot reimburse, the bank will give her the times to repay, she becomes a debtor, she moves from state B to state D with probability 1-β. If she fails to reimburse again, the bank will give her opportunities again to repay, as she stays at the state D with probability ε. On the contrary, if she repays her property, she returns to state A with probability 1-ε and may request another property. The rules can be summarized in a Markov chain $(X_t)_{t\in N}$ with the set of states S:=$\{A,B,D\}$, where $X_{t\in S}$. The model can be illustrated by the following picture:

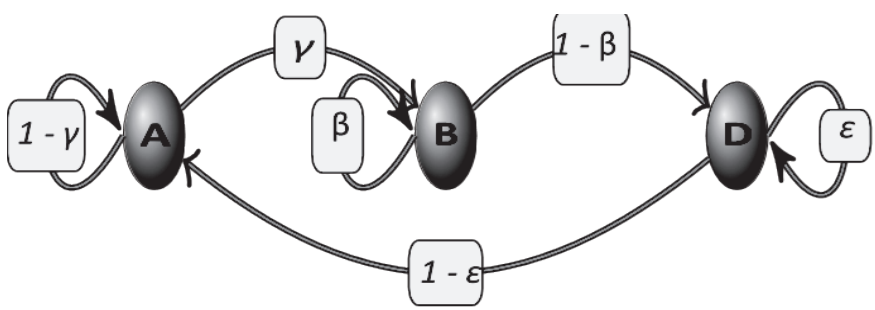

Figure 2.
Markov Chain Model of *Murabahah* Contract

We can also describe the model into the transition matrix P_r:

$$P_r = \begin{pmatrix} 1-\gamma & \gamma & 0 \\ 0 & \beta & 1-\beta \\ 1-\varepsilon & 0 & \varepsilon \end{pmatrix}.$$

The beneficiary receives the goods from the bank with the price K. The beneficiary must pay the bank, $K + s$. The amount of s depends on the contract as the bank profits.

In the *murabahah* contract, we consider the financial flow function g:SxS→R. The beneficiary profits are the goods, which the beneficiary received. The beneficiary profits for one period in a *murabahah* contract are $g(A, B)$ or $g(B, B)$, signifying the beneficiary gaining profit when she passes state A to state B or when she passes state B to state B.

The bank's profits for one period in a *murabahah* contract is

$$g_k(X_{t-1}, X_t) = \begin{cases} s, & if\ (B,B) \\ s, & if\ (D,A) \\ 0, & if\ not. \end{cases}$$

The expected total profit of bank at state x and the time t in *murabahah* contract is

$$V_k(x) = \sum_{t\geq 1} \delta^t\, g_k(X_{t-1}, X_t),$$

where δ is the discount for the time of one loan.

3.2.3. Proportion of Beneficiaries in The Population at Equilibrium

An interesting consequence of the Markov chain for modeling of the dynamics is the study of the evolution of population distribution in various states, specifically the influence of the various parameters of the model to the proportion of beneficiaries in a population at equilibrium. We can use two propositions as applications of Perron-Frobenius theorem to calculate the proposition of each state in the population at equilibrium.

Proposition 1.

Let $(X_t)_{t\geq 1}$ be above the Markov chain of mudarabah with the set of states S:=$\{A,B\}$ and transition matrix P_d. The Markov chain has a unique equilibrium state π^*,

$$\pi^* = \frac{1}{\gamma+\varepsilon}(\varepsilon, \gamma).$$

The proportions of the two states tend to the equilibrium distribution π^* when t tends ∞.

We can use this proposition to calculate the proportion of A and B. The proportion of A (π_A^*) $= \frac{\varepsilon}{\gamma+\varepsilon}$ and the proportion of B (π_B^*) $= \frac{\gamma}{\gamma+\varepsilon}$.

Example:

The number of clients of *mudarabah* contract in a bank is 500. If γ=0.5 and ε=0.1, then the proportion of clients at state A = 0.17 (83 applicants) and the proportion of clients at state B = 0.83 (417 beneficiaries).

On the other hand, drawing on the Perron-Frobenius theorem, we can estimate certain parameters of the Markov chain model by using a historical loan of the beneficiary for many period of the time t. The bank can use the data to calculate the proportion of every state base on the Ergodic theorem, then we can estimate γ and ε.

Proposition 2.

Let $(X_t)_{t\geq 1}$ be above the Markov chain of murabahah with the set of states $S:=\{A,B,D\}$ and transition matrix P_r. The Markov chain has a unique equilibrium state π^*,

$$\pi^* = \frac{1}{(1-\beta)(1-\varepsilon)+\gamma(1-\varepsilon)+\gamma(1-\beta)}((1-\beta)(1-\varepsilon), \gamma(1-\varepsilon), \gamma(1-\beta)).$$

The proportions of the two states tend to the equilibrium distribution π^* when t tends ∞.

From this proposition, the proportion of A $(\pi_A^*) = \frac{(1-\beta)(1-\varepsilon)}{(1-\beta)(1-\varepsilon)+\gamma(1-\varepsilon)+\gamma(1-\beta)}$, the proportion of B $(\pi_B^*) = \frac{\gamma(1-\varepsilon)}{(1-\beta)(1-\varepsilon)+\gamma(1-\varepsilon)+\gamma(1-\beta)}$, and the proportion of D $(\pi_D^*) = \frac{\gamma(1-\beta)}{(1-\beta)(1-\varepsilon)+\gamma(1-\varepsilon)+\gamma(1-\beta)}$.

Example:

The number of clients of *murabahah* contract in a bank is 500. If γ=0.8, β=0.7, and ε=0.1, then the proportion of clients at state A = 0.22 (110 applicants), the proportion of clients at state B = 0.58 (293 beneficiaries), and the proportion of clients at state D = 0.28 (97 debtors).

By using the Perron-Frobenius theorem and the Ergodic theorem, we can estimate γ, ε, and β:

$$\frac{1-\beta}{\gamma} = \frac{\pi_A^*}{\pi_B^*}, \qquad \frac{1-\varepsilon}{1-\beta} = \frac{\pi_B^*}{\pi_A^*}, \qquad \text{and } \frac{1-\varepsilon}{\gamma} = \frac{\pi_A^*}{\pi_D^*}.$$

3.2.4. Absence of Strategic Default

We take an idea introduced by Tedeschi (2006) which be called the absence of strategic default. Tedeschi explains a way to minimize the strategic default (moral hazard) by the beneficiary. We propose a mechanism that assures repayment in a *mudabarah* contract. It must be certain that the expected total profit of beneficiary when she can reimburse her loan must be more than the expected total profit of beneficiary when she cannot reimburse her loan and the beneficiary must reimburse the total money.

$$W_l(B) \geq W_l(A) + \left[K + \rho\big(Y(K) - (c + K)\big)\right]. \tag{3}$$

Furthermore, we can see a constraint of the parameters and study the consequences. We apply a proposition, which is proposed by Nahla et al. (2013)

about calculating the expected total profit of the beneficiary to see the value of the parameters, which respect the constraint of the absence of strategic default.

$$E\big(W(x)\big) = \delta(I - \delta P)^{-1} Z(x). \tag{4}$$

Where I is the identity matrix, P is the transition matrix, and $Z(x)=E(f(X_0,X_1) \mid X_0=x)$.

IV. RESULTS AND ANALYSIS
4.1. Results
4.1.1. Simulation of Number of Loans
4.1.1.1. Simulation of Number of Loans in *Mudarabah* Contract

In a contract credit, both the bank and beneficiary expect that the beneficiary gets successful to reimburse her loan, so that the beneficiary can get a new loan. Such dynamics are expected to occur repeatedly.

In the *mudarabah* contract, we must maximize her proportion of B to maximize the number of loans. From proposition 1, $\pi_B^* = \frac{\gamma}{\gamma+\varepsilon}$, we must increase γ and/or decrease ε to a get large proportion of B. We decide to choose different values of γ in our simulations. In practice, the bank determines γ (the bank determines the number of applicants who will be accepted as a beneficiary).

We simulate different γ, γ=0.5 and γ=0.8, and we also choose ε=0.1, with the number of applicants is 500 and the number of periods t is 24. For each values of γ, we do 10 simulations to estimate the number of loans in a *mudarabah* contract, and then we calculate the average of each case.

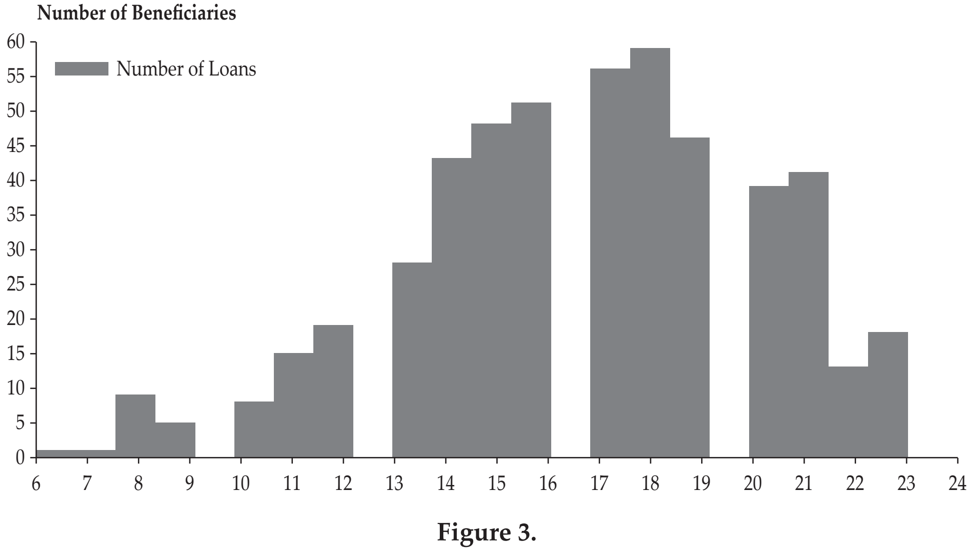

Figure 3.
Distribution of Number of Loans in *Mudarabah* Contract for γ=0.5

The first case γ=0.5, it means the number of applicants who become the beneficiary is 250. From the simulation, we get an average of modus of the number of loans is 18. It is obtained by 60 beneficiaries or 24.00% of the number of beneficiaries.

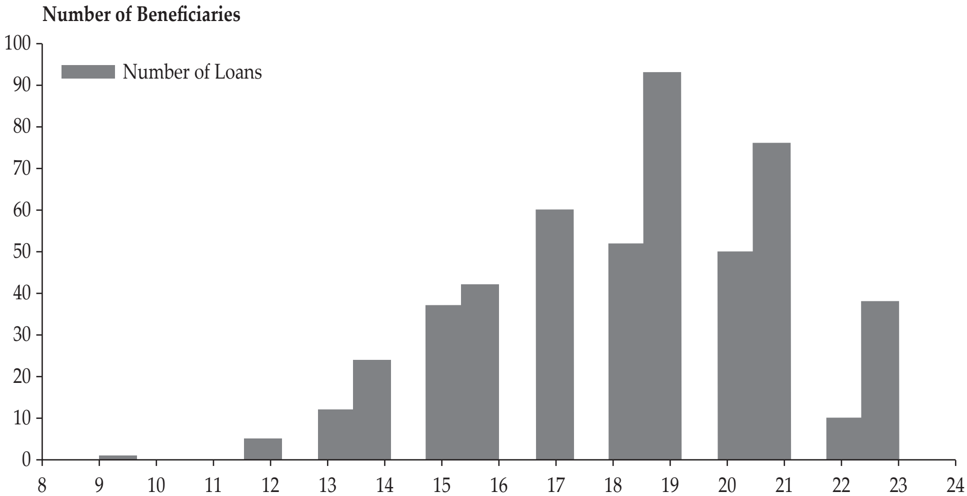

Figure 4.
Distribution of Number of Loans in *Mudarabah* Contract for γ=0.8

The second case, γ=0.8 (the number of applicants who become the beneficiary is 400), we get an average of the modus of the number of loans is 19, obtained by 94 beneficiaries (23.50% of beneficiaries).

Table 1.
Comparison the Distributions of Number of Loans in *Mudarabah* Contract for Two Different Values of γ

γ	Number of Loans	Number of Beneficiaries	% of Beneficiaries
0.5	18	60	24.00
0.8	19	94	23.50

4.1.1.2. Simulation of Number of Loans in *Murabahah* Contract

In the *murabahah* model, the bank and the beneficiary hope, the beneficiary stays in state B, so we must maximize the proportion of B. From the proposition 2, the proportion of B is: $\frac{\gamma(1-\varepsilon)}{(1-\beta)(1-\varepsilon)+\gamma(1-\varepsilon)+\gamma(1-\beta)} = \frac{1}{\frac{1-\beta}{\gamma}+1+\frac{1-\beta}{1-\varepsilon}}$, so we must increase γ and/or β, and/or decrease ε to get a large proportion of B.

Similar to the previous simulations in *mudarabah* model, we choose different γ, such as γ=0.5 and γ=0.8. We also choose ε=0.1, with the number of applicants is 500 and the number of periods t is 24. We perform 10 simulations for each γ to estimate the number of loans in *murabahah* contract.

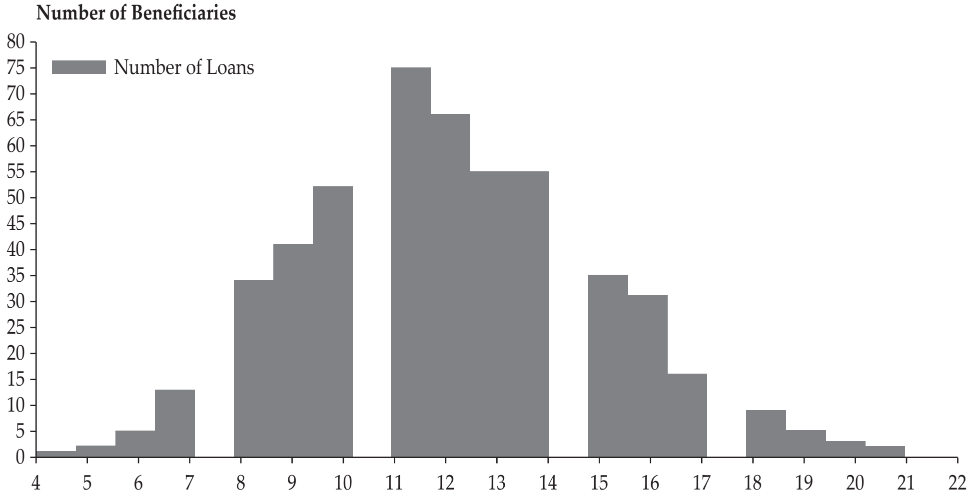

Figure 5.
Distribution of Number of Loans in *Murabahah* Contract for γ=0.5

In the case γ=0.5, we get an average of the modus of the number of loans is 11, it is obtained by 73 beneficiaries (29.20% of beneficiaries).

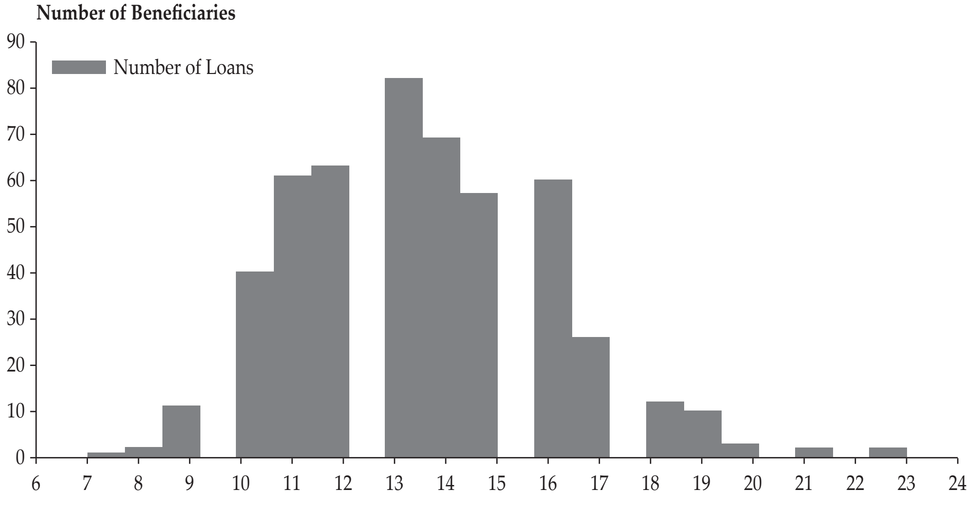

Figure 6.
Distribution of Number of Loans in *Murabahah* Contract for γ=0.8

In the case γ=0.8, we get an average of the modus of the number of loans is 13, it is obtained by 82 beneficiaries (20.50% of beneficiaries).

Table 2.
Comparison the Distributions of Number of Loans in *Murabahah* Contract for Two Different Values of γ

γ	Maximum Number of Loans	Number of Beneficiaries	% of Beneficiaries
0.5	11	73	29.20
0.8	13	82	20.50

4.1.2. Simulation of the Expected Total Profit

4.1.2.1. Simulation of the Expected Total Profit in *Mudarabah* Contract

We want to approximate the expected total profit of the beneficiary and the expected total profit of the bank in the *mudarabah* contract. We perform some simulations with the number of applicants is 500 and the number of periods t is 24. We do 10 simulations for each of two different values of γ (γ=0.5 and γ=0.8). The other parameters that we choose, ε=0.1, δ=0.9, ρ=0.3, K = RM100000, c = RM5000. If the beneficiary gets profit, $Y(K)$ = RM145000, while if the beneficiary lost, $Y(K)$ = RM85000.

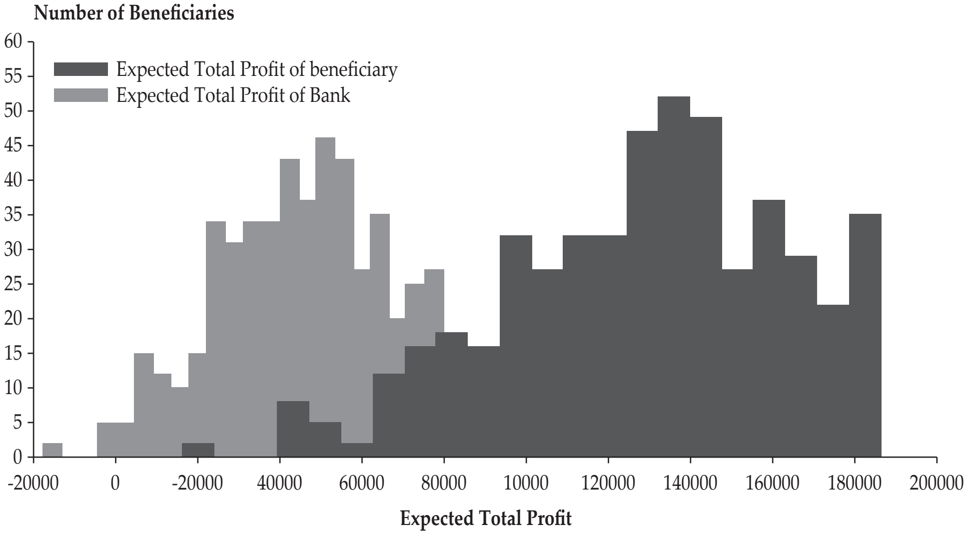

Figure 7.
Distribution of the Expected Total Profit in *Mudarabah* Contract for γ=0.5

For γ=0.5, the modus of the expected total profit of the beneficiary is RM134500, obtained by 31 beneficiaries (20.40% of beneficiaries), and the modus of the expected total profit of the bank is RM53500 that the bank obtains from 46 beneficiaries (18.40% of beneficiaries).

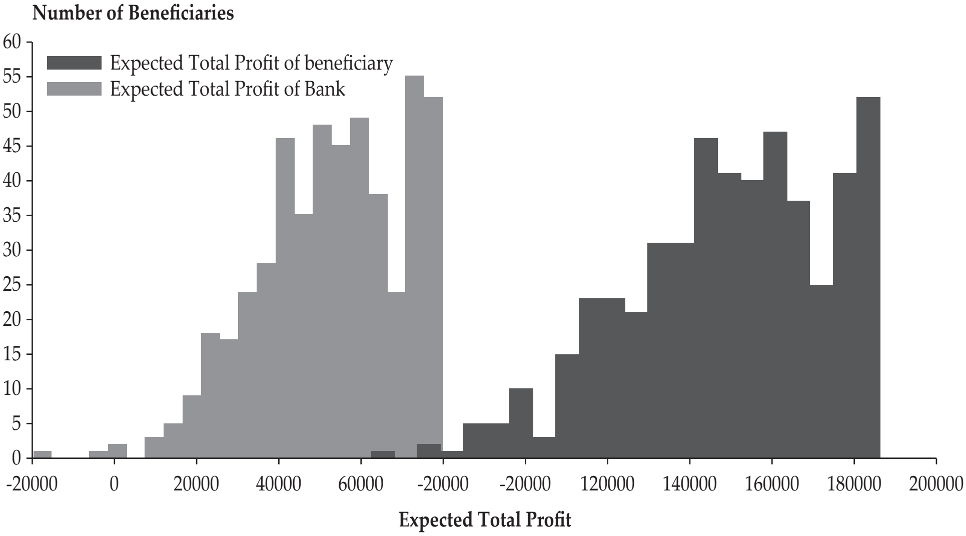

Figure 8.
Distribution of the Expected Total Profit in *Mudarabah* Contract for γ=0.8

For γ=0.8, the results are the average of modus of the expected total profit of the beneficiary is RM180000, obtained by 55 beneficiaries (13.75% of beneficiaries), and the average of modus of the expected total profit of the bank is RM72500 that the bank obtains from 56 beneficiaries (14.00% of beneficiaries).

Table 3. Comparison the Distributions of the Expected Total Profit in *Mudarabah* Contract for Two Different Values of γ

	Bank			Beneficiary		
γ	Expected Total Profit (RM)	Number of Beneficiaries	% of Beneficiaries	Expected Total Profit (RM)	Number of Beneficiaries	% of Beneficiaries
0.5	53500	46	18.40	134500	51	20.40
0.8	72500	56	14.00	180000	55	13.75

4.1.2.2. Simulation of the Expected Total Profit in *Murabahah* Contract

We make some simulations, which similar to the simulations of *mudarabah* contract, but only to estimate the expected total profit of the bank. We choose two different values of γ (γ=0.5 and γ=0.8), ε=0.1, β=0.7, δ=0.9, K = \$100000, s = \$20000, the number of applicants is 500, and the number of periods t is 24.

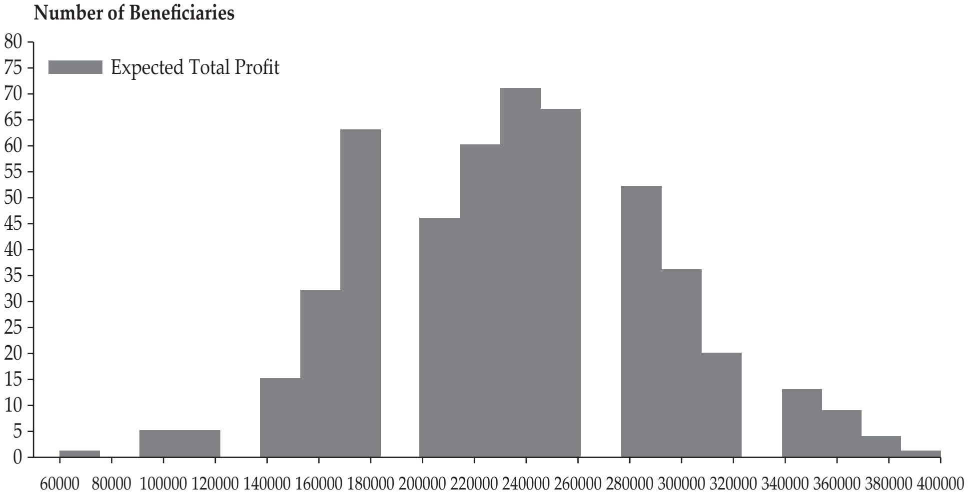

Figure 9.
Distribution of the Expected Total Profit in *Murabahah* Contract for γ=0.5

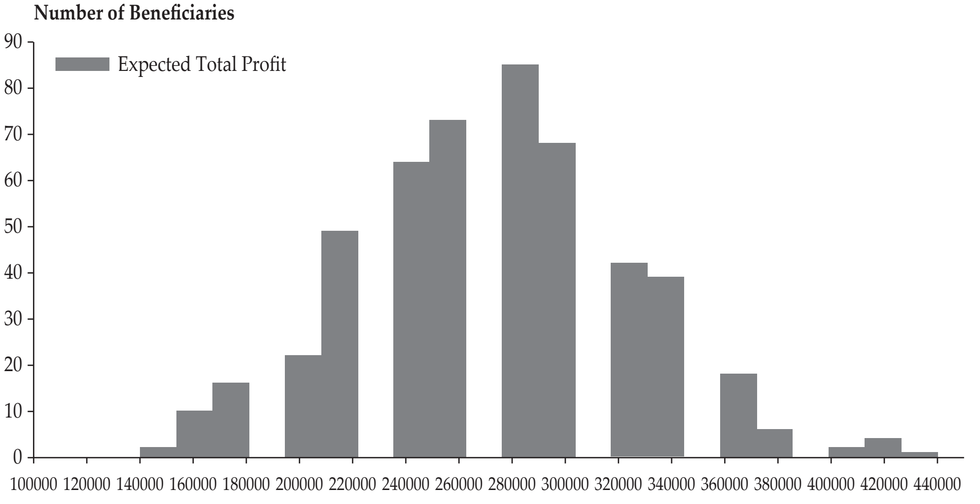

Figure 10.
Distribution of the Expected Total Profit in *Murabahah* Contract for γ=0.8

The simulations results for γ=0.5 the average of modus of the expected total profit of the bank is $233000, that the bank obtains from 71 beneficiaries (28.40% of the number of beneficiaries), and for γ=0.8, the average of modus of the expected total profit of the bank is $281000, from 85 beneficiaries (21.25% of beneficiaries).

Table 4.
Comparison the Distributions of the Expected Total Profit in *Murabahah* Contract for Two Different Values of γ

γ	Expected Total Profit ($)	Number of Beneficiaries	% of Beneficiaries
0.5	233000	71	28.40
0.8	281000	85	21.25

4.1.3. Relation Between the Bank's Portion of Profits and the Probability of Risk in *Mudarabah* Contract

In a *mudarabah* contract, to avoid the risk due to adverse selection, the bank opts to determine the bank's portion of profits (ρ), which benefits the bank. Of course, it is not desired by the beneficiary. It is important that the bank must find another solution for the problem.

In our model, if the expected total profit of the bank equal to the expected total profit of the beneficiary, we get a relation between the bank's portion of profit and the probability of risk in a *mudarabah* contract. The bank can choose ρ to avoid or reduce the risk (ε). By using the equation (2), the bank can choose ρ to estimate ε.

We can describe the relationship between ρ and ε with the following picture:

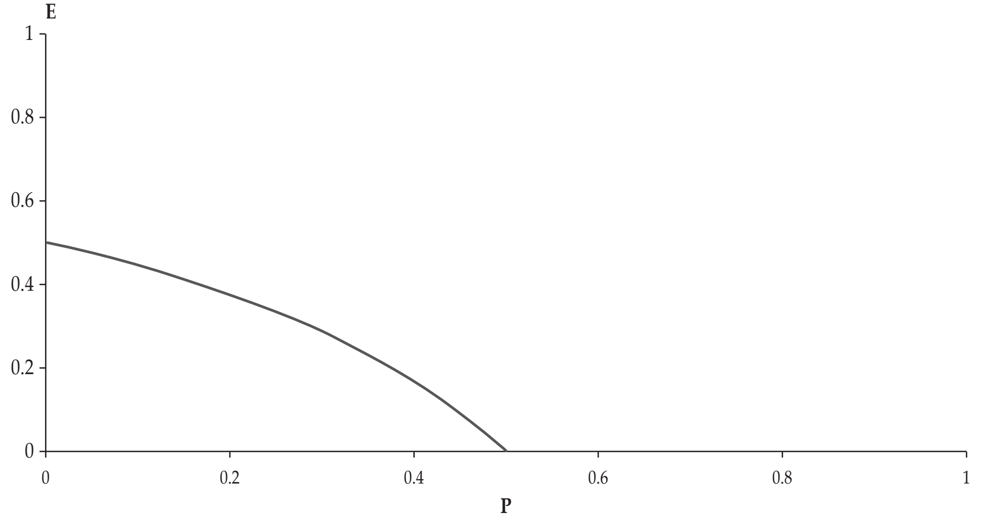

Figure 11.
The Relation between the Bank's Portion of Profits and the Probability of Risk

4.1.4. Absence of Strategic Default

We can apply formulas (3) and (4) to find the constraints of the probability of risk (ε) at the condition of the absence of strategic default in *mudarabah* contract. To avoid or reduce the risk due to strategic default (moral hazard), we want to find the constraints of ε at the condition of the absence of strategic default. We search the constraints of ε in two different values of γ (γ=0.5 and γ=0.8). We choose K = RM100000, Y(K) = RM140000, c = RM5000, δ=0.9, and ρ=0.3.

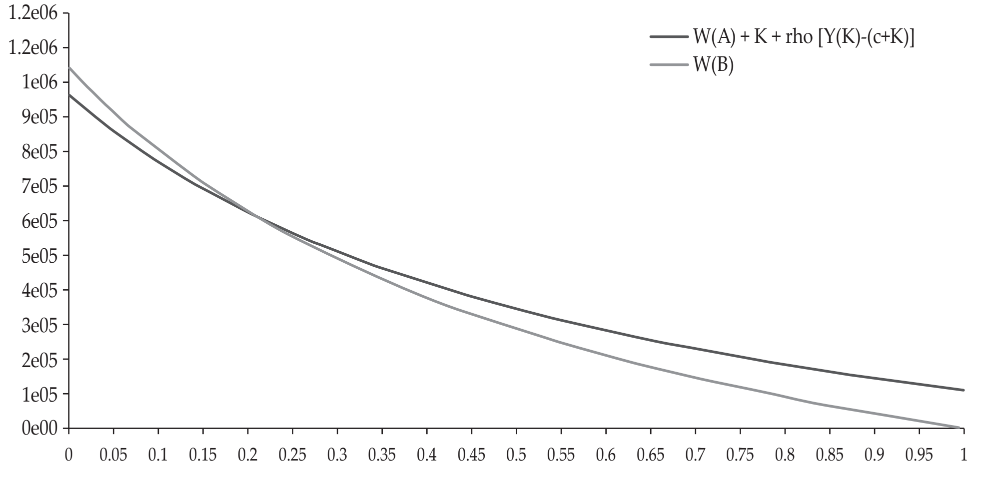

Figure 12.
The Constraint of the Probability of Risk at Absence of Strategic Default for γ=0.5

In the first case, γ=0.5, we find the constraints of ε at the condition of absence of strategic default is 0.20.

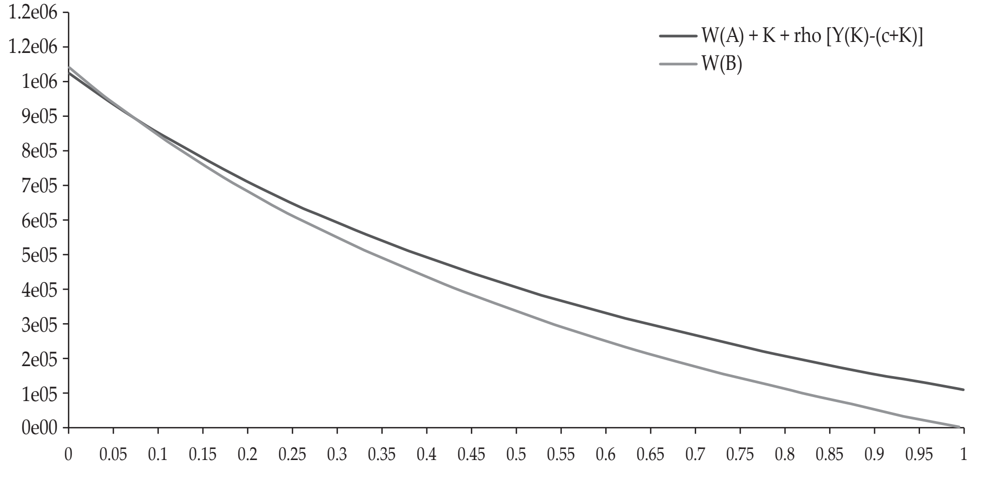

Figure 13.
The Constraint of the Probability of Risk at Absence of Strategic Default for γ=0.8

In the second case (γ=0.8), we find the constraint of ε at the condition of absence of strategic default is 0.05. Accordingly, there is a greater chance of strategic default for this case.

Table 5. Comparison of the Constraints of the Risk at the Condition of Absence of Strategic Default for Two Different Values of γ

γ	Constraints of ε
0.5	0.20
0.8	0.05

4.2. Analysis

In the simulations of the number of loans, if we compare the results of simulations of two different values of γ in *mudarabah* and *murabahah* contract (table 1 and table 2), we can see that the percentage of beneficiaries gets the maximum number of loans, be greater when γ=0.5. It means the Islamic bank can decrease the probability of the applicant to be a beneficiary to increase the number of loans. By maximizing the number of loans, Islamic bank can ensure to reduce the risk due to moral hazard.

The results of the simulations of the expected total profit in *mudarabah* and *murabahah* contract (Tables 3 and 4) show that the Islamic bank can decrease the probability of beneficiary applicants to increase the percentage of the number of beneficiaries who get the optimal profits in *mudarabah* contract and increase the percentage of number of beneficiaries who give the optimal profits to the bank in *mudarabah* and *murabahah* contracts. It can also assure reducing the risk due to moral hazard.

The Islamic bank must be more careful in selecting customers to reduce the number of applicants as the beneficiary to avoid adverse selection, so the Islamic bank can give more chance to beneficiaries to receive the credit to increase their profit, ultimately reducing the risk due to moral hazard.

In the *mudarabah* contract, we can see the relation between the bank's portion of profit and the probability of risk in Fig. 11. If the bank's portion of profit increases than the probability of risk decreases and vice versa. If the Islamic bank chooses a large bank's portion of profit, then the number of applicants decreases because the applicants' hope the bank's portion of the profit is small. This causes the risk of decreasing adverse selection or moral hazard. On the contrary, if the bank chooses a small portion of profit, the number of applicants could increase, causing the risk due to increased adverse selection or moral hazard.

Based on the absence of strategic default, we can see the comparison of the constraints of the risk at the condition of absence of strategic default for two different values of the probability of the applicant to be a beneficiary. If the bank increases the probability of the applicant to be a beneficiary, then the constraint of the probability of risk at the condition of the absence of strategic default decreases. As a result, the probability of risk due to decreased strategic default (moral hazard), when the bank decreases the probability of the applicant be a beneficiary.

V. CONCLUSION AND RECOMMENDATION

5.1. Conclusion

We have introduced the Markov chain model of Islamic micro-financing, especially in *mudarabah* and *murabahah* contracts. By using the models, the Islamic bank can determine the expected total profit of the beneficiary and the expected total profit of the bank and calculate the proportion of beneficiaries of each state at equilibrium condition. The Islamic bank can also formulate the condition of the absence of strategic default to avoid or reduce the risk in *mudarabah* contract. The Islamic bank can decrease the probability of the applicant be a beneficiary to avoid or reduce the adverse selection or moral hazard.

5.2. Recommendation

The Islamic bank must be more selective in selecting the beneficiary to avoid or reduce the risk due to adverse selection and moral hazard, as well as to optimize the profit of the beneficiary and the profit of bank.

Further research could discuss a formula of the condition of absence of strategic default to avoid the risk in *murabahah* contract. We also propose to use the Markov Chain model in the other types of the contract of Islamic micro-financing to study the risk due to adverse selection and moral hazard.

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